Astron. Nachr. **269**, 48 (1939)

The asymmetry in the distribution of sunspots over the solar disk

By M. Minnaert, Utrecht

Abstract

1. A graphical interpretation is given, from which the cause of the East-West asymmetry in the appearance of sunspots becomes immediatley clear.
2. The distribution of the emergence of sunspots with heliographic longitude is derived.
3. We consider the entire curve of evolution of sunspots; from this, in accordance with observations, we derive that more sunspots disappear on the West than on the East.
4. We point out that statistical analyses of the East-West asymmetry need to be divided into 2 groups: one regarding emergences and disappearances of spots, and one regarding the overall numbers and areas of spots. The first group of asymmetries results clearly from simple theory, the second necessarily brings us to the hypothesis that a sunspot of a given size is more visible in the Eastern half than the Western half of the Sun.

In Astr. Nachr 263.13 and 266.81, Gleissberg discussed the facts already repeatedly noted that more sunspots are born on the Eastern half of the Sun than on the Western one[1]. The explanation he proposed is the same as the one given by Schuster in 1911[2], and consists of the following two hypotheses: 1. each group develops slowly and can be observed only if it reaches a known minimum size; 2. to be visible on the limb of the solar disk a sunspot has to have a larger area than on the center; more precisely Gleissberg sets this threshold size to be directly proportional to $\sec\epsilon$, where ϵ = distance from the center.

1. Let us introduce a simple diagram, from which the nature of this explanation will become clear (Figure 1). To simplify things, let us consider a sunspot on the solar equator and assume that the rotation axis is perpendicular to the ecliptic. On the x axis is the heliographic longitude l, with zero on the Central Meridian; the region between -90° and $+90^\circ$ corresponds to the visible half of the Sun. On the y-axis is the area of the sunspot group. Let us assume that groups are born along AB at a fixed distance from each other, and that their areas grow linearly with time. Therefore we obtain a set of parallel straight lines, of constant inclination determined by the speed of sunspot development.

As soon as it reaches the threshold area, a group becomes visible. The threshold size in the range -90° to $+90^\circ$ is given by curve CDE, which according to Gleissberg has the form $\sec l$. A group becomes visible where it crosses the curve CDE.

One can see immediately that more groups become visible between C and D than between D and E. Therefore one can see that beyond the heliographic longitude l_m no group can become visible. At the same time one finds that there is a maximum value, l^*, of the heliographic longitude at which sunspots seen from Earth are formed. The spots formed at longitudes greater

[1] A.S.D. Maunder, MNRAS 65, 451 (1907)
[2] A. Schuster, Proc Roy Soc London A 85, 309 (1911)

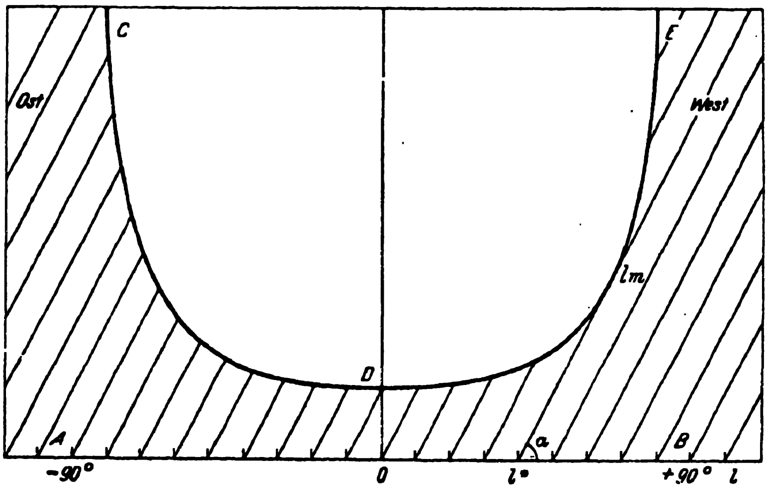

Figure 1: The asymmetry in the appearence of new sunspots, for the simple case of a straight line development curve.

than l^* can become infinitely big without being seen; this conclusion however becomes illusory as soon as we take into account the finite lifetime of sunspots. To the West of l^* all sunspot groups become invisible even before they cross the limb.

Figure 1 is also useful if the sunspot is found outside the Equator. Only, in the latter case, it is not $\epsilon = l$, but $\cos\epsilon = \cos l \, \cos b$ where b is the heliographic latitude and the limit of visibility is given by $1/(\cos l \, \cos b)$.

2. For a complete statistics it may be interesting not only to discuss the relationship between the total count of new spots on the East and on the West, but also the distribution of formation over heliographic longitude.

While the visibility threshold is $y = \frac{1}{\cos l \, \cos b}$, its inclination is $\frac{dy}{dl} = \frac{\sin l}{\cos^2 l \, \cos b}$; the angle of inclination is $\phi = \arctan\left(\frac{\sin l}{\cos^2 l \, \cos b}\right)$.

Let α be the angle characterising the lines of growth in area of sunspots (see Figure 1). Therefore these lines meet the threshold curve with an angle α-ϕ; the density of sunspot formation at heliolongitude l becomes:

$$\frac{dn}{dl} = \frac{\sin(\alpha - \phi)}{\cos\alpha \, \sin\phi} = 1 - \frac{\tan\alpha}{\tan\phi} = 1 - \frac{\sin l}{\cos^2 l \, \cos b \, \tan\alpha} \tag{1}$$

Figure 2 gives the curves of density of spot formation for different values of α, using for $\cos b$ a mean value of 0.97. According to the statistics derived by Schuster, it shows how many new groups are born in each longitude interval. One can notice how the general process that gives rise to the East-West asymmetry of newly formed spots is clearly demonstrated.

We note that our curves are valid only in the limit that the evolution of the sunspots in the first part of their lifetime takes place at a constant speed.

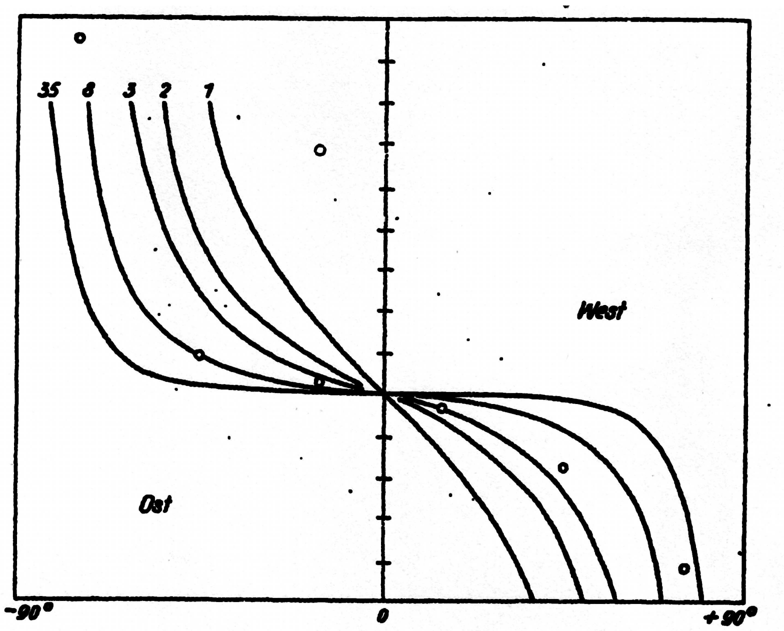

Figure 2: The distribution of newly emerged sunspots over heliographic longitude, for different values of the speed of their growth in area ($\tan\alpha$= 1, 2, 3, 8, 35).

3. At first sight it appears surprising that from a simple statistics of visible sunspots one arrives to a discussion of the evolution of the period of visibility. In reality even this is not the case. The only thing that can be deduced from an ideal, complete and exact distribution is the inclination of the curve of evolution in every point where they cross the visibility curve; one can determine after how many days a 2nd, 3rd, 4th, ... n-th sunspot has become so large, until the time when it becomes visible in the middle of the disk. The distribution does not say anything about what has happened before. But one can make the hypothesis that the sunspots have developed from a zero area and with the same speed with which we see them grow later on. Really from the distribution it can simply be verified that 'after the sunspot has become visible in the middle of the disk it takes 0.3 days for it to double its size'. From this, Schuster and Gleissberg conclude not without reason that the sunspot must have grown approximately 0.3 days before we can observe its appearing in the middle of the disk.

The same result could be derived also from a statistics of the average increase in sunspot areas. Approximately this is of the order of 100 millionths of the visible solar hemisphere per day.[3] But unfortunately it changes so fast in the first day of life of the sunspot, that the real time of formation cannot be calculated from this. Instead, it is interesting to link this speed of development with the time, established by Gleissberg, of 0.3 days between the formation and the appearing of the sunspot. During this time the area of a large sunspot must have grown by about 30 millionths of solar hemisphere, that is it must have grown to a radius of 5 arcsec before it can be detected. This number could seem quite large but it should be noted that here we are considering the time at which a group of sunspots is recognised as such, not the maximum area

[3] MNRAS, 85, 553 (1925)

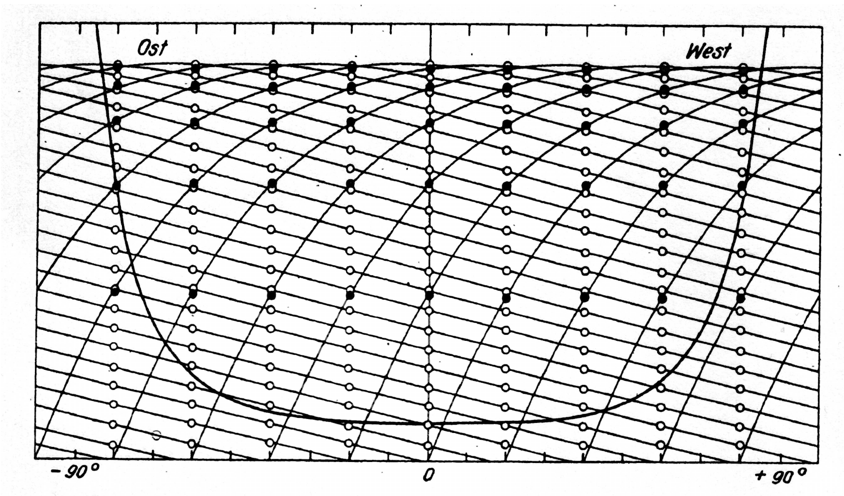

Figure 3: The distribution of spots that emerge and disappear, for the curve of evolution of 30-day sunspots.

of the smallest visible details on the disk.

4. Finally, we can try to use the complete curve of growth as observed, of a sunspot that has grown and disappeared, to derive the sunspot distribution. In this way, not only we could hope to obtain a more satisfactory description of the evolution, but also the disappearance of spots or groups of spots would be understood from theory. By means of our graphical method this problem can be easily described (Figure 3). As an example we use the growth curve of 30-day spots as given in Figure 4.

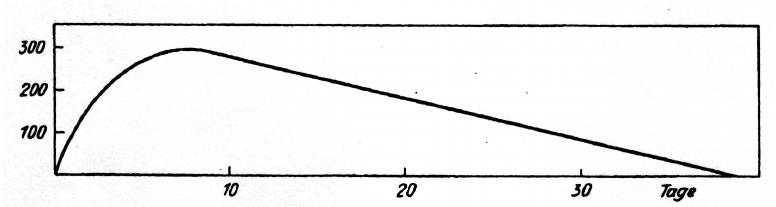

Figure 4: Average development curve for a 30-day sunspot. The y axis gives area in millionths of the solar hemisphere.

The following observations would be expected:

a) In the heliolongitude bin between $-90°$ and $-84°$ no spots would be visible. Between $-84°$ and $-43°$ the growth of spots should be well observed, but not the disappearance of spots. Between $-43°$ and $+73°$, they should be seen both emerging and disappearing. Between $+73°$ and $+84°$ only disappearance should be observed. Beyond $+84°$ no spots are visible. (The numbers given are valid only for the growth curve we used here).

b) The number of spots that disappear is larger on the western half than on the eastern half. This conclusion is in agreement with statistical results and can be seen in the Figure above.

c) On the Eastern side, spots don't appear in the order of their real birth. The location of appearence runs backwards towards the East, then it moves towards the West, and spots that are growing appear simultaneously with spots whose area is decreasing.

From Figure 3 it can be seen that the conclusions obtained regarding the birth and disappearance of spots are by and large independent of the specific form of the curve of evolution or of the visibility curve over the solar disk, which may not be entirely proportional to $\sec l$. Every change in the curve of sunspot evolution or visibility function will influence the quantitative values, but it will hardly affect the general process. The agreement with observations is hardly surprising.

5. It is more surprising that a second group of statistical observations exists that refers to the distribution of the total number of spots (not just newly formed spots) and of their areas, in different heliographic longitudes, and which cannot be explained by theories such as those of Schuster and Gleissberg. Mrs Maunder for example found that the eastern hemisphere always has more spots than the western one. In our derivation we assumed that in any heliographic longitude several spots are born in succession and precisely (for simplicity) at the same distance from each other. In Figure 3, the spots that are growing are indicated with a filled circle, the ones that are decaying with an empty circle. Looking at Figure 3 one can immediately see the perfect symmetry of the spot distribution despite the strong asymmetry of the growth curve. One can also see that no asymmetry in the areas can develop between East and West.

If we assume that the data of Mrs Maunder are sufficiently accurate, we are forced to change our interpretation in the following way:

a) Either, the net of the development curve may not cover the Figure in all places in the same way, and this would be the case if the Earth has an influence on the spots and their growth curve, and these are different on one half of the Sun than on the other.

b) Or, the visibility curve itself is asymmetric. Mrs Maunder put forward the possibility that a spot may be more visible on the East than on the West, for example if the axes of the spots are not perpendicular to the surface of the Sun.

Our graphical interpretation shows that there are no other possibilities. The second hypothesis is given preference. Before this can be considered accurate it will be necessary to get more information and modern data.

Translated from German by Rosanna Modena Dalla and Silvia Dalla; 27/12/2006. (Please email any suggestions for improvements in this translation to Silvia Dalla: sdalla@uclan.ac.uk).