Box/peanut-shaped bulges in action space: supplementary figures

Victor P. Debattista\textsuperscript{1*}, David J. Liddicott\textsuperscript{1}, Tigran Khachaturyants\textsuperscript{1}, Leandro Beraldo e Silva\textsuperscript{1}
\textsuperscript{1} Jeremiah Horrocks Institute, University of Central Lancashire, Preston, PR1 2HE, UK

APPENDIX A: MODEL 1

The initial conditions of model 1 are identical to those of the fiducial model 2, but cooler, with $\sigma_{R0} = 90$ km s$^{-1}$ instead of 128 km s$^{-1}$. In this appendix we present the figures equivalent to Figs. 3 to 12 for model 2.

APPENDIX B: MODEL 3

The initial conditions of model 3 are identical to those of the fiducial model 2, but hotter, with $\sigma_{R0} = 165$ km s$^{-1}$ instead of 128 km s$^{-1}$. In this appendix we present the figures equivalent to Figs. 3 to 12 for model 2.

APPENDIX C: MODEL 4

The initial conditions of model 4 are identical to those of the fiducial model 2, but thinner, with $z_d = 150$ pc instead of 300 pc. In this appendix we present the figures equivalent to Figs. 3 to 12 for model 2.

APPENDIX D: MODEL 5

The initial conditions of model 5 are identical to those of the fiducial model 2, but thicker, with $z_d = 600$ pc instead of 300 pc. In this appendix we present the figures equivalent to Figs. 3 to 12 for model 2.

APPENDIX E: MODEL T1

The initial conditions of model T1 consist of a cool thin disc and a hot thick disc, with surface mass density similar to model 2. In this appendix we present the figures equivalent to Figs. 3 to 12 for model 2.

APPENDIX F: MODEL T5

The initial conditions of model T5 consist of a hot thin disc and a hot thick disc, with surface mass density similar to model 2. In this appendix we present the figures equivalent to Figs. 3 to 12 for model 2.

APPENDIX G: MODEL HD1

The initial conditions of model HD1 consist of a dark matter-dominated system with a Toomre-$Q$ profile similar to that of model 1. In this appendix we present the figures equivalent to Figs. 3 to 12 for model 2.

APPENDIX H: MODEL HD2

The initial conditions of model HD2 consist of a dark matter-dominated system with a Toomre-$Q$ profile similar to that of model 2. In this appendix we present the figures equivalent to Figs. 3 to 12 for model 2.
Figure A1. The density distribution of model 1 at $t = 0$ (left) and at the end of the simulation, at $t = 5$ Gyr, (right) for 3 different projections. The density levels are arbitrary. A prominent bar, with a B/P-shaped bulge, has formed by $t = 5$ Gyr. The bar has been rotated into the $x$-axis.
Figure A2. Evolution of the mass distribution in model 1 separated into action quartiles. At top is $J_{\phi,0}$, followed by $J_{x,0}$, while at bottom is $J_{y,0}$. The first and third columns show the lowest quartile while the second and fourth columns show the highest quartile. The two left columns show $t = 0$, while the two right columns show the end of the simulation, at 5 Gyr.
Figure A3. The bar strength, \( a_2(J) \), in model 1 as a function of the initial actions. Only bins with more than 200 particles are shown.

Figure A4. The bar strength as a function of radius for model 1 split into different action quartiles. The top row shows the \( m = 2 \) Fourier amplitudes while the bottom row shows the \( m = 4 \) ones. At left the disc is split into \( J_{R,0} \) quartiles while at right into \( J_{z,0} \) quartiles.
Figure A5. The face-on surface density (white contours) and mean actions (colours and black contours) at $t = 0$ (left) and at $t = 5$ Gyr (right) in model 1.
Figure A6. Actions in the \((x, z)\) plane at \(t = 0\) (left) and at the end of the simulation, at \(t = 5\) Gyr (middle) and in the \((y, z)\) plane (right) in model 1. From top to bottom we plot \(\langle J_\phi, 0 \rangle\), \(\langle J_R, 0 \rangle\) and \(\langle J_z, 0 \rangle\). Particles are selected in the slice \(|y| < 2\) kpc in the two left columns and in the slice \(|z| < 2\) kpc in the right column. Black contours show the actions, while white contours show the density.
Figure A7. The change in the vertical profiles of $\langle J_{\phi,0} \rangle$ (top row), $\langle J_{z,0} \rangle$ (second row), $\langle J_{R,0} \rangle$ (third row), and $\langle R_0 \rangle$ (fourth row) in model 1. The initial conditions are shown as (red) filled circles while the final distribution is shown as (black) filled squares. All quantities have been computed within the inner $R_d = 2.4 \text{ kpc}$.

Figure A8. The central vertical density profile of stars in model 1 at $t = 5 \text{ Gyr}$ split into two equal samples of low and high actions, as indicated. The full sample of stars is shown by the black line. The profiles have been computed within the inner $R_d = 2.4 \text{ kpc}$. 
Debattista et al.

Figure A9. Density profiles along the $x$-axis (along which the bar is aligned) in model 1 at the end of the simulation, $t = 5$ Gyr, for different heights above the mid-plane. At each height, the profiles are split into quartiles of the initial actions, as indicated in the top row, with the 1st quartile having the lowest values. Only particles at $|y| < 2$ kpc are included. From left to right profiles are split by the azimuthal, radial and vertical actions.
Figure A10. Left: Evolution of $h_z$, the thickness of stellar populations, in the $(J_{R,0}, J_{z,0})$ plane of model 1. Right: Evolution of $\sigma_R$, the radial random motion of stellar populations, in the $(J_{R,0}, J_{z,0})$ plane. The top row shows $t = 0$ and the middle row at the end of the simulation, at $t = 5$ Gyr, both on a log scale. The bottom row shows the difference, now on a linear scale. At large $J_{z,0}$, $\Delta h_z < 0$, i.e., the stellar populations become thinner after the B/P bulge forms (bottom left). Similarly, at large $J_{R,0}$, $\Delta \sigma_R < 0$, i.e., the stellar populations become radially cooler (bottom right). The dashed grey contours indicate $\Delta h_z = 0$ (bottom left) and $\Delta \sigma_R = 0$ (bottom right). Solid contours indicate the density of stars and are spaced by factors of 10, with the peak set at 10,000 particles.
Figure B1. The density distribution of model 3 at $t = 0$ (left) and at the end of the simulation, at $t = 5$ Gyr, (right) for 3 different projections. The density levels are arbitrary. A prominent bar, with a B/P-shaped bulge, has formed by $t = 5$ Gyr. The bar has been rotated into the $x$-axis.
Figure B2. Evolution of the mass distribution in model 3 separated into action quartiles. At top is $J_{z,0}$, followed by $J_{\phi,0}$, while at bottom is $J_{\nu,0}$. The first and third columns show the lowest quartile while the second and fourth columns show the highest quartile. The two left columns show $t=0$, while the two right columns show the end of the simulation, at 5 Gyr.
Figure B3. The bar strength, $a_2(J)$, in model 3 as a function of the initial actions. Only bins with more than 200 particles are shown.

Figure B4. The bar strength as a function of radius for model 3 split into different action quartiles. The top row shows the $m = 2$ Fourier amplitudes while the bottom row shows the $m = 4$ ones. At left the disc is split into $J_{R,0}$ quartiles while at right into $J_{z,0}$ quartiles.
Figure B5. The face-on surface density (white contours) and mean actions (colours and black contours) at $t = 0$ (left) and at $t = 5$ Gyr (right) in model 3.
**Figure B6.** Actions in the \((x, z)\) plane at \(t = 0\) (left) and at the end of the simulation, at \(t = 5\) Gyr (middle) and in the \((y, z)\) plane (right) in model 3. From top to bottom we plot \(\langle J_\phi, 0 \rangle\), \(\langle J_R, 0 \rangle\) and \(\langle J_z, 0 \rangle\). Particles are selected in the slice \(|y| < 2\) kpc in the two left columns and in the slice \(|x| < 2\) kpc in the right column. Black contours show the actions, while white contours show the density.
Figure B7. The change in the vertical profiles of $\langle J_{φ,0} \rangle$ (top row), $\langle J_{z,0} \rangle$ (second row), $\langle J_{R,0} \rangle$ (third row), and $\langle R_{0} \rangle$ (fourth row) in model 3. The initial conditions are shown as (red) filled circles while the final distribution is shown as (black) filled squares. All quantities have been computed within the inner $R_d = 2.4$ kpc.

Figure B8. The central vertical density profile of stars in model 3 at $t = 5$ Gyr split into two equal samples of low and high actions, as indicated. The full sample of stars is shown by the black line. The profiles have been computed within the inner $R_d = 2.4$ kpc.
Figure B9. Density profiles along the x-axis (along which the bar is aligned) in model 3 at the end of the simulation, $t = 5$ Gyr, for different heights above the mid-plane. At each height, the profiles are split into quartiles of the initial actions, as indicated in the top row, with the 1st quartile having the lowest values. Only particles at $|y| < 2 \text{kpc}$ are included. From left to right are shown the azimuthal, radial and vertical actions.
Figure B10. Left: Evolution of $h_z$, the thickness of stellar populations, in the $(J_{R,0}, J_z,0)$ plane of model 3. Right: Evolution of $\sigma_R$, the radial random motion of stellar populations, in the $(J_{R,0}, J_z,0)$ plane. The top row shows $t = 0$ and the middle row at the end of the simulation, at $t = 5$ Gyr, both on a log scale. The bottom row shows the difference, now on a linear scale. At large $J_{z,0}$, $\Delta h_z < 0$, i.e. the stellar populations become thinner after the B/P bulge forms (bottom left). Similarly, at large $J_{R,0}$, $\Delta \sigma_R < 0$, i.e. the stellar populations become radially cooler (bottom right). The dashed grey contours indicate $\Delta h_z = 0$ (bottom left) and $\Delta \sigma_R = 0$ (bottom right). Solid contours indicate the density of stars and are spaced by factors of 10, with the peak set at 10,000 particles.
Figure C1. The density distribution of model 4 at $t = 0$ (left) and at the end of the simulation, at $t = 5$ Gyr, (right) for 3 different projections. The density levels are arbitrary. A prominent bar, with a B/P-shaped bulge, has formed by $t = 5$ Gyr. The bar has been rotated into the $x$-axis.
Figure C2. Evolution of the mass distribution in model 4 separated into action quartiles. At top is $J_{1,0}$, followed by $J_{2,0}$, while at bottom is $J_{0,0}$. The first and third columns show the lowest quartile while the second and fourth columns show the highest quartile. The two left columns show $t = 0$, while the two right columns show the end of the simulation, at 5 Gyr.
Figure C3. The bar strength, $a_2(J)$, in model 4 as a function of the initial actions. Only bins with more than 200 particles are shown.

Figure C4. The bar strength as a function of radius for model 4 split into different action quartiles. The top row shows the $m = 2$ Fourier amplitudes while the bottom row shows the $m = 4$ ones. At left the disc is split into $J_{R,0}$ quartiles while at right into $J_{z,0}$ quartiles.
Figure C5. The face-on surface density (white contours) and mean actions (colours and black contours) at \( t = 0 \) (left) and at \( t = 5 \) Gyr (right) in model 4.
Figure C6. Actions in the $(x, z)$ plane at $t = 0$ (left) and at the end of the simulation, at $t = 5$ Gyr (middle) and in the $(y, z)$ plane (right) in model 4. From top to bottom we plot $\langle J_{\phi,0} \rangle$, $\langle J_{R,0} \rangle$ and $\langle J_{z,0} \rangle$. Particles are selected in the slice $|y| < 2$ kpc in the two left columns and in the slice $|x| < 2$ kpc in the right column. Black contours show the actions, while white contours show the density.
Figure C7. The change in the vertical profiles of $\langle J_{\phi,0} \rangle$ (top row), $\langle J_{z,0} \rangle$ (second row), $\langle J_{R,0} \rangle$ (third row), and $\langle R_0 \rangle$ (fourth row) in model 4. The initial conditions are shown as (red) filled circles while the final distribution is shown as (black) filled squares. All quantities have been computed within the inner $R_d = 2.4$ kpc.

Figure C8. The central vertical density profile of stars in model 4 at $t = 5$ Gyr split into two equal samples of low and high actions, as indicated. The full sample of stars is shown by the black line. The profiles have been computed within the inner $R_d = 2.4$ kpc.
Figure C9. Density profiles along the x-axis (along which the bar is aligned) in model 4 at the end of the simulation, \( t = 5 \) Gyr, for different heights above the mid-plane. At each height, the profiles are split into quartiles of the initial actions, as indicated in the top row, with the 1st quartile having the lowest values. Only particles at \(|y| < 2 \) kpc are included. From left to right are shown the azimuthal, radial and vertical actions.
Figure C10. Left: Evolution of $h_z$, the thickness of stellar populations, in the $(J_{R,0}, J_{z,0})$ plane of model 4. Right: Evolution of $\sigma_R$, the radial random motion of stellar populations, in the $(J_{R,0}, J_{z,0})$ plane. The top row shows $t = 0$ and the middle row at the end of the simulation, at $t = 5$ Gyr, both on a log scale. The bottom row shows the difference, now on a linear scale. At large $J_{R,0}$, $\Delta h_z < 0$, i.e. the stellar populations become thinner after the B/P bulge forms (bottom left). Similarly, at large $J_{R,0}$, $\Delta \sigma_R < 0$, i.e. the stellar populations become radially cooler (bottom right). The dashed grey contours indicate $\Delta h_z = 0$ (bottom left) and $\Delta \sigma_R = 0$ (bottom right). Solid contours indicate the density of stars and are spaced by factors of 10, with the peak set at 10,000 particles.

© — RAS, MNRAS 000, 1–1
Figure D1. The density distribution of model 5 at $t = 0$ (left) and at the end of the simulation, at $t = 5$ Gyr, (right) for 3 different projections. The density levels are arbitrary. A prominent bar, with a B/P-shaped bulge, has formed by $t = 5$ Gyr. The bar has been rotated into the $x$-axis.
Figure D2. Evolution of the mass distribution in model 5 separated into action quartiles. At top is $J_{\phi,0}$, followed by $J_{z,0}$, while at bottom is $J_{R,0}$. The first and third columns show the lowest quartile while the second and fourth columns show the highest quartile. The two left columns show $t = 0$, while the two right columns show the end of the simulation, at 5 Gyr.
Figure D3. The bar strength, $a_2(J)$, in model 5 as a function of the initial actions. Only bins with more than 200 particles are shown.

Figure D4. The bar strength as a function of radius for model 5 split into different action quartiles. The top row shows the $m = 2$ Fourier amplitudes while the bottom row shows the $m = 4$ ones. At left the disc is split into $J_{R,0}$ quartiles while at right into $J_{z,0}$ quartiles.
Figure D5. The face-on surface density (white contours) and mean actions (colours and black contours) at $t = 0$ (left) and at $t = 5$ Gyr (right) in model 5.
Figure D6. Actions in the $(x, z)$ plane at $t = 0$ (left) and at the end of the simulation, at $t = 5$ Gyr (middle) and in the $(y, z)$ plane (right) in model 5. From top to bottom we plot $\langle J_\phi, 0 \rangle$, $\langle J_R, 0 \rangle$ and $\langle J_z, 0 \rangle$. Particles are selected in the slice $|y| < 2$ kpc in the two left columns and in the slice $|x| < 2$ kpc in the right column. Black contours show the actions, while white contours show the density.
Figure D7. The change in the vertical profiles of $\langle J_{\phi}, 0 \rangle$ (top row), $\langle J_z, 0 \rangle$ (second row), $\langle J_R, 0 \rangle$ (third row), and $\langle R_0 \rangle$ (fourth row) in model 5. The initial conditions are shown as (red) filled circles while the final distribution is shown as (black) filled squares. All quantities have been computed within the inner $R_d = 2.4$ kpc.

Figure D8. The central vertical density profile of stars in model 5 at $t = 5$ Gyr split into two equal samples of low and high actions, as indicated. The full sample of stars is shown by the black line. The profiles have been computed within the inner $R_d = 2.4$ kpc.
Debattista et al.

Figure D9. Density profiles along the $x$-axis (along which the bar is aligned) in model 5 at the end of the simulation, $t = 5$ Gyr, for different heights above the mid-plane. At each height, the profiles are split into quartiles of the initial actions, as indicated in the top row, with the 1st quartile having the lowest values. Only particles at $|y| < 2$ kpc are included. From left to right are shown the azimuthal, radial and vertical actions.
Figure D10. Left: Evolution of $h_z$, the thickness of stellar populations, in the ($J_{R,0}, J_{z,0}$) plane of model 5. Right: Evolution of $\sigma_R$, the radial random motion of stellar populations, in the ($J_{R,0}, J_{z,0}$) plane. The top row shows $t = 0$ and the middle row at the end of the simulation, at $t = 5$ Gyr, both on a log scale. The bottom row shows the difference, now on a linear scale. At large $J_{z,0}$, $\Delta h_z < 0$, i.e. the stellar populations become thinner after the B/P bulge forms (bottom left). Similarly, at large $J_{R,0}$, $\Delta \sigma_R < 0$, i.e. the stellar populations become radially cooler (bottom right). The dashed grey contours indicate $\Delta h_z = 0$ (bottom left) and $\Delta \sigma_R = 0$ (bottom right). Solid contours indicate the density of stars and are spaced by factors of 10, with the peak set at 10,000 particles.
Figure E1. The density distribution of model T1 at $t = 0$ (left) and at the end of the simulation, at $t = 5$ Gyr, (right) for 3 different projections. The density levels are arbitrary. A prominent bar, with a B/P-shaped bulge, has formed by $t = 5$ Gyr. The bar has been rotated into the $x$-axis.
Figure E2. Evolution of the mass distribution in model T1 separated into action quartiles. At top is $J_{R,0}$, followed by $J_{z,0}$, while at bottom is $J_{\phi,0}$. The first and third columns show the lowest quartile while the second and fourth columns show the highest quartile. The two left columns show $t = 0$, while the two right columns show the end of the simulation, at 5 Gyr.
Figure E3. The bar strength, $a_2(J)$, in model T1 as a function of the initial actions. Only bins with more than 200 particles are shown.

Figure E4. The bar strength as a function of radius for model T1 split into different action quartiles. The top row shows the $m = 2$ Fourier amplitudes while the bottom row shows the $m = 4$ ones. At left the disc is split into $J_{R,0}$ quartiles while at right into $J_{z,0}$ quartiles.
Figure E5. The face-on surface density (white contours) and mean actions (colours and black contours) at $t = 0$ (left) and at $t = 5$ Gyr (right) in model T1.
Figure E6. Actions in the $(x, z)$ plane at $t = 0$ (left) and at the end of the simulation, at $t = 5$ Gyr (middle) and in the $(y, z)$ plane (right) in model T1. From top to bottom we plot $\langle J_{\phi,0} \rangle$, $\langle J_{R,0} \rangle$ and $\langle J_{z,0} \rangle$. Particles are selected in the slice $|y| < 2$ kpc in the two left columns and in the slice $|x| < 2$ kpc in the right column. Black contours show the actions, while white contours show the density.
**Figure E7.** The change in the vertical profiles of $\langle J_{\phi,0} \rangle$ (top row), $\langle J_{z,0} \rangle$ (second row), $\langle J_{R,0} \rangle$ (third row), and $\langle R_0 \rangle$ (fourth row) in model T1. The initial conditions are shown as (red) filled circles while the final distribution is shown as (black) filled squares. All quantities have been computed within the inner $R_d = 2.4$ kpc.

**Figure E8.** The central vertical density profile of stars in model T1 at $t = 5$ Gyr split into two equal samples of low and high actions, as indicated. The full sample of stars is shown by the black line. The profiles have been computed within the inner $R_d = 2.4$ kpc.
Debattista et al.

Figure E9. Density profiles along the $x$-axis (along which the bar is aligned) in model T1 at the end of the simulation, $t = 5$ Gyr, for different heights above the mid-plane. At each height, the profiles are split into quartiles of the initial actions, as indicated in the top row, with the 1st quartile having the lowest values. Only particles at $|y| < 2$ kpc are included. From left to right are shown the azimuthal, radial and vertical actions.
Figure E10. Left: Evolution of $h_z$, the thickness of stellar populations, in the ($J_{R0}, J_{z0}$) plane of model T1. Right: Evolution of $\sigma_R$, the radial random motion of stellar populations, in the ($J_{R0}, J_{z0}$) plane. The top row shows $t = 0$ and the middle row at the end of the simulation, at $t = 5$ Gyr, both on a log scale. The bottom row shows the difference, now on a linear scale. At large $J_{z0}$, $\Delta h_z < 0$, i.e. the stellar populations become thinner after the B/P bulge forms (bottom left). Similarly, at large $J_{R0}$, $\Delta \sigma_R < 0$, i.e. the stellar populations become radially cooler (bottom right). The dashed grey contours indicate $\Delta h_z = 0$ (bottom left) and $\Delta \sigma_R = 0$ (bottom right). Solid contours indicate the density of stars and are spaced by factors of 10, with the peak set at 10,000 particles.
Figure F1. The density distribution of model T5 at $t = 0$ (left) and at the end of the simulation, at $t = 5$ Gyr, (right) for 3 different projections. The density levels are arbitrary. A prominent bar, with a B/P-shaped bulge, has formed by $t = 5$ Gyr. The bar has been rotated into the $x$-axis.
Figure F2. Evolution of the mass distribution in model T5 separated into action quartiles. At top is $J_{R,0}$, followed by $J_{z,0}$, while at bottom is $J_{\phi,0}$. The first and third columns show the lowest quartile while the second and fourth columns show the highest quartile. The two left columns show $t = 0$, while the two right columns show the end of the simulation, at 5 Gyr.
Figure F3. The bar strength, \(\alpha_2(J)\), in model T5 as a function of the initial actions. Only bins with more than 200 particles are shown.

Figure F4. The bar strength as a function of radius for model T5 split into different action quartiles. The top row shows the \(m = 2\) Fourier amplitudes while the bottom row shows the \(m = 4\) ones. At left the disc is split into \(J_{R,0}\) quartiles while at right into \(J_{z,0}\) quartiles.
Figure F5. The face-on surface density (white contours) and mean actions (colours and black contours) at $t = 0$ (left) and at $t = 5$ Gyr (right) in model T5.
Figure F6. Actions in the \((x, z)\) plane at \(t = 0\) (left) and at the end of the simulation, at \(t = 5\) Gyr (middle) and in the \((y, z)\) plane (right) in model T5. From top to bottom we plot \(\langle J_{\phi, 0}\rangle\), \(\langle J_{R, 0}\rangle\) and \(\langle J_{z, 0}\rangle\). Particles are selected in the slice \(|y| < 2\) kpc in the two left columns and in the slice \(|x| < 2\) kpc in the right column. Black contours show the actions, while white contours show the density.
Figure F7. The change in the vertical profiles of \(\langle J_{\phi,0} \rangle\) (top row), \(\langle J_{z,0} \rangle\) (second row), \(\langle J_{R,0} \rangle\) (third row), and \(\langle R_0 \rangle\) (fourth row) in model T5. The initial conditions are shown as (red) filled circles while the final distribution is shown as (black) filled squares. All quantities have been computed within the inner \(R_d = 2.4\) kpc.

Figure F8. The central vertical density profile of stars in model T5 at \(t = 5\) Gyr split into two equal samples of low and high actions, as indicated. The full sample of stars is shown by the black line. The profiles have been computed within the inner \(R_d = 2.4\) kpc.
Figure F9. Density profiles along the $x$-axis (along which the bar is aligned) in model T5 at the end of the simulation, $t = 5$ Gyr, for different heights above the mid-plane. At each height, the profiles are split into quartiles of the initial actions, as indicated in the top row, with the 1st quartile having the lowest values. Only particles at $|y| < 2$ kpc are included. From left to right are shown the azimuthal, radial and vertical actions.
Figure F10. Left: Evolution of $h_z$, the thickness of stellar populations, in the $(J_{R,0}, J_{z,0})$ plane of model T5. Right: Evolution of $\sigma_R$, the radial random motion of stellar populations, in the $(J_{R,0}, J_{z,0})$ plane. The top row shows $t = 0$ and the middle row at the end of the simulation, at $t = 5$ Gyr, both on a log scale. The bottom row shows the difference, now on a linear scale. At large $J_{z,0}$, $\Delta h_z < 0$, i.e. the stellar populations become thinner after the B/P bulge forms (bottom left). Similarly, at large $J_{R,0}$, $\Delta \sigma_R < 0$, i.e. the stellar populations become radially cooler (bottom right). The dashed grey contours indicate $\Delta h_z = 0$ (bottom left) and $\Delta \sigma_R = 0$ (bottom right). Solid contours indicate the density of stars and are spaced by factors of 10, with the peak set at 10,000 particles.
Figure G1. The density distribution of model HD1 at $t = 0$ (left) and at the end of the simulation, at $t = 10$ Gyr, (right) for 3 different projections. The density levels are arbitrary. A prominent bar, with a B/P-shaped bulge, has formed by $t = 10$ Gyr. The bar has been rotated into the $x$-axis.
Figure G2. Evolution of the mass distribution in model HD1 separated into action quartiles. At top is $J_{R,0}$, followed by $J_{z,0}$, while at bottom is $J_{\phi,0}$. The first and third columns show the lowest quartile while the second and fourth columns show the highest quartile. The two left columns show $t = 0$, while the two right columns show the end of the simulation, at 10 Gyr.
Figure G3. The bar strength, $a_2(J)$, in model HD1 as a function of the initial actions. Only bins with more than 200 particles are shown.

Figure G4. The bar strength as a function of radius for model HD1 split into different action quartiles. The top row shows the $m = 2$ Fourier amplitudes while the bottom row shows the $m = 4$ ones. At left the disc is split into $J_R,0$ quartiles while at right into $J_{x,0}$ quartiles.
Figure G5. The face-on surface density (white contours) and mean actions (colours and black contours) at $t = 0$ (left) and at $t = 10$ Gyr (right) in model HD1.
Figure G6. Actions in the \((x, z)\) plane at \(t = 0\) (left) and at the end of the simulation, at \(t = 10\) Gyr (middle) and in the \((y, z)\) plane (right) in model HD1. From top to bottom we plot \(\langle J_\phi, 0 \rangle\), \(\langle J_R, 0 \rangle\) and \(\langle J_z, 0 \rangle\). Particles are selected in the slice \(|y| < 2\) kpc in the two left columns and in the slice \(|x| < 2\) kpc in the right column. Black contours show the actions, while white contours show the density.
Figure G7. The change in the vertical profiles of \( \langle J_{\varphi,0} \rangle \) (top row), \( \langle J_{z,0} \rangle \) (second row), \( \langle J_{R,0} \rangle \) (third row), and \( \langle R_0 \rangle \) (fourth row) in model HD1. The initial conditions are shown as (red) filled circles while the final distribution is shown as (black) filled squares. All quantities have been computed within the inner \( R_d = 2.4 \) kpc.

Figure G8. The central vertical density profile of stars in model HD1 at \( t = 10 \) Gyr split into two equal samples of low and high actions, as indicated. The full sample of stars is shown by the black line. The profiles have been computed within the inner \( R_d = 2.4 \) kpc.
Figure G9. Density profiles along the $x$-axis (along which the bar is aligned) in model HD1 at the end of the simulation, $t = 10$ Gyr, for different heights above the mid-plane. At each height, the profiles are split into quartiles of the initial actions, as indicated in the top row, with the 1st quartile having the lowest values. Only particles at $|y| < 2$ kpc are included. From left to right are shown the azimuthal, radial and vertical actions.
Figure G10. Left: Evolution of $h_z$, the thickness of stellar populations, in the $(J_{R,0}, J_{z,0})$ plane of model HD1. Right: Evolution of $\sigma_R$, the radial random motion of stellar populations, in the $(J_{R,0}, J_{z,0})$ plane. The top row shows $t = 0$ and the middle row at the end of the simulation, at $t = 10$ Gyr, both on a log scale. The bottom row shows the difference, now on a linear scale. At large $J_{z,0}$, $\Delta h_z < 0$, i.e. the stellar populations become thinner after the B/P bulge forms (bottom left). Similarly, at large $J_{R,0}$, $\Delta \sigma_R < 0$, i.e. the stellar populations become radially cooler (bottom right). The dashed grey contours indicate $\Delta h_z = 0$ (bottom left) and $\Delta \sigma_R = 0$ (bottom right). Solid contours indicate the density of stars and are spaced by factors of 10, with the peak set at 10,000 particles.
© RAS, MNRAS 000, 1–1
Figure H1. The density distribution of model HD2 at $t = 0$ (left) and at the end of the simulation, at $t = 10$ Gyr, (right) for 3 different projections. The density levels are arbitrary. A prominent bar, with a B/P-shaped bulge, has formed by $t = 10$ Gyr. The bar has been rotated into the $x$-axis.
Figure H2. Evolution of the mass distribution in model HD2 separated into action quartiles. At top is $J_{R,0}$, followed by $J_{z,0}$, while at bottom is $J_{\phi,0}$. The first and third columns show the lowest quartile while the second and fourth columns show the highest quartile. The two left columns show $t = 0$, while the two right columns show the end of the simulation, at 10 Gyr.
Figure H3. The bar strength, $a_2(J)$, in model HD2 as a function of the initial actions. Only bins with more than 200 particles are shown.

Figure H4. The bar strength as a function of radius for model HD2 split into different action quartiles. The top row shows the $m = 2$ Fourier amplitudes while the bottom row shows the $m = 4$ ones. At left the disc is split into $J_{R,0}$ quartiles while at right into $J_{z,0}$ quartiles.
Figure H5. The face-on surface density (white contours) and mean actions (colours and black contours) at $t = 0$ (left) and at $t = 10$ Gyr (right) in model HD2.
Figure H6. Actions in the $\langle x, z \rangle$ plane at $t = 0$ (left) and at the end of the simulation, at $t = 10$ Gyr (middle) and in the $\langle y, z \rangle$ plane (right) in model HD2. From top to bottom we plot $\langle J_\phi, 0 \rangle$, $\langle J_R, 0 \rangle$ and $\langle J_z, 0 \rangle$. Particles are selected in the slice $|y| < 2$ kpc in the two left columns and in the slice $|z| < 2$ kpc in the right column. Black contours show the actions, while white contours show the density.
Figure H7. The change in the vertical profiles of $\langle J_\phi, 0 \rangle$ (top row), $\langle J_z, 0 \rangle$ (second row), $\langle J_R, 0 \rangle$ (third row), and $\langle R_0 \rangle$ (fourth row) in model HD2. The initial conditions are shown as (red) filled circles while the final distribution is shown as (black) filled squares. All quantities have been computed within the inner $R_d = 2.4$ kpc.

Figure H8. The central vertical density profile of stars in model HD2 at $t = 10$ Gyr split into two equal samples of low and high actions, as indicated. The full sample of stars is shown by the black line. The profiles have been computed within the inner $R_d = 2.4$ kpc.
Figure H9. Density profiles along the $x$-axis (along which the bar is aligned) in model HD2 at the end of the simulation, $t = 10$ Gyr, for different heights above the mid-plane. At each height, the profiles are split into quartiles of the initial actions, as indicated in the top row, with the 1st quartile having the lowest values. Only particles at $|y| < 2$ kpc are included. From left to right are shown the azimuthal, radial and vertical actions.
Figure H10. Left: Evolution of $h_z$, the thickness of stellar populations, in the $(J_{R,0}, J_{z,0})$ plane of model HD2. Right: Evolution of $\sigma_R$, the radial random motion of stellar populations, in the $(J_{R,0}, J_{z,0})$ plane. The top row shows $t = 0$ and the middle row at the end of the simulation, at $t = 10$ Gyr, both on a log scale. The bottom row shows the difference, now on a linear scale. At large $J_{z,0}$, $\Delta h_z < 0$, i.e. the stellar populations become thinner after the B/P bulge forms (bottom left). Similarly, at large $J_{R,0}$, $\Delta \sigma_R < 0$, i.e. the stellar populations become radially cooler (bottom right). The dashed grey contours indicate $\Delta h_z = 0$ (bottom left) and $\Delta \sigma_R = 0$ (bottom right). Solid contours indicate the density of stars and are spaced by factors of 10, with the peak set at 10,000 particles.