On the arduous task of modelling rotating A-type stars and their pulsations

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Why would anyone want to do that ? !

(Modelling rotating A-type stars and their pulsations)

- Non-standard stellar physics related to convective cores and rotation:
  - transport and mixing phenomena due to rotation,
  - mixing due to overshooting,
  - baroclinic flows,
  - turbulence,
  - diffusion,
  -...
- Advances in understanding these phenomena will allow to interprete observations fruitfully
- Unprecedented asteroseismic potential which results are hindered by our lack of knowledge of these phenomena

1. Expected rotational effects on A-type stars structure

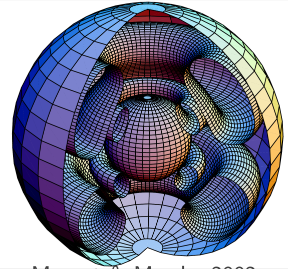
2. Impact of rotation on their pulsations

3. Asteroseismic inferences of rotation

What rotation does to stars

Meridional Circulation in radiative envelopes

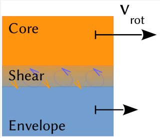
Zahn 1992, Maeder & Zahn 1998



Meynet & Maeder 2002

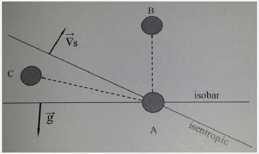
Shear induced turbulence

Chaboyer & Zahn 1992



Baroclinic instabilities

Mathis et al. 2004



→ Transport of chemical elements (evolution !), and angular momentum

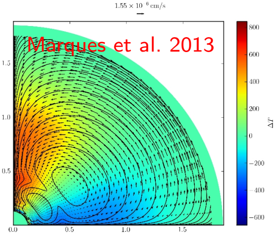
Stellar evolution modelling with rotation

In spherical symmetry

YREC (Pinsonnault 1988, Chaboyer 1995),
Geneva Evolution code (Talon et al. 1997),
STAREVOL (Palacios et al. 2003),
CESTAM (Marques et al. 2013)

Success e.g. Li depletion in A-F stars
(Charbonnel & Talon 1999)

Meridional Circulation



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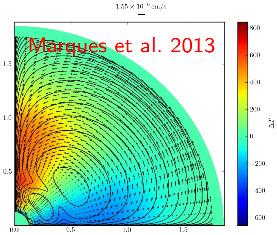
In two dimensions (steady state)

- Evolution in spherical symmetry
- Ad hoc rotation profile
- Hydrostatic Equilibrium including centrifugal force

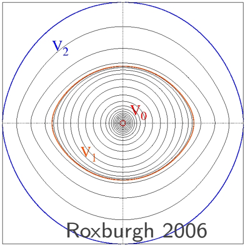
SCF method (Jackson et al. 2004 2005)
Characteristics method (Roxburgh 2004 2006)

→ Asteroseismic and interferrometric obs^{o}

Meridional Circulation



Centrifugal Distortion



Stellar evolution modelling with rotation

In two dimensions (dynamical)

The ESTER project: 2D hydro simulations
(Espinosa Lara & Rieutord 2007...2013)

- So far nuclear evolution ad hoc
- Convective core (isentropic), no convective envelope M $\gtrsim 1.5 M_{\odot}$
- Differential rotation as produced by Baroclinic torques in radiative zone (RZ)

Stellar evolution modelling with rotation

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- So far nuclear evolution ad hoc
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- Differential rotation as produced by Baroclinic torques in radiative zone (RZ)

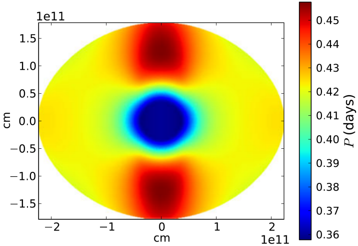
→ Fast core rotating as a cylinder, shellular rotation in the inner part of the RZ, latitudinal differential in the outer part.

→ Improvement of gravity darkening law

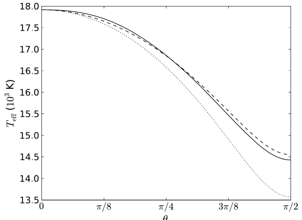
→ http://ester-project.github.io/ester/

→ See talk G. Halabi

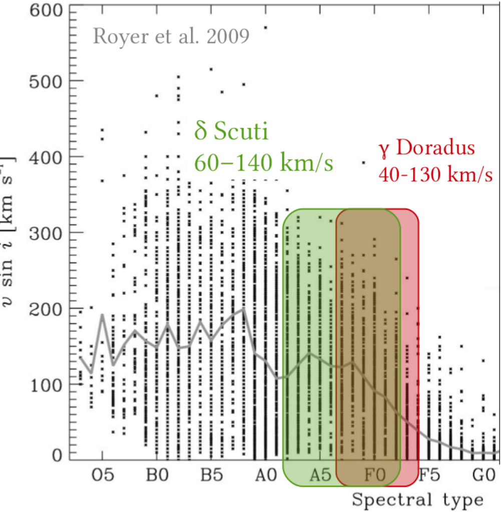
Rotation map
Rieutord & Espinosa Lara 2013



Gravity darkening law
Espinosa Lara & Rieutord 2012



Impact of rotation on A-stars pulsations



→ Talk by Frédéric Royer

through:

- Centrifugal force $\propto r\Omega^2$

Distorts mainly the outer envelope

→ p-modes in δ Scuti stars

- Coriolis force $\propto \Omega v$

Affects the pulsation dynamics, important when $P_{rot} \sim P_{puls}$

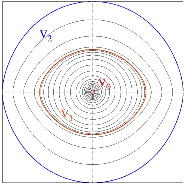
→ g-modes in γ Doradus stars

roAp stars

Rapidly oscillating Ap stars

→ Talks by J. Matthews, D. Gough, M. Cunha, H. Saio and P. Quitral-Manosalva

Ouazzani et al. 2015



Effect of fast rotation on δScuti's pulsations

- Through a 2-dimensional, non-perturbative treatment

Reese et al 2006, 2009b, Ouazzani et al. 2012b, 2015

- Account for the full influence of the Coriolis force:

$$\frac{\partial \mathbf{v}'}{\partial t} + (\mathbf{v_0} \cdot \nabla)\mathbf{v}' + 2\Omega \times \mathbf{v}' + (\mathbf{v}' \cdot \nabla)\mathbf{v_0} = -\frac{1}{\rho_0}\nabla p' - \nabla\Phi' + \frac{\rho'}{\rho_0^2} p_0$$

Non-separability of the equations system in terms of r and (θ,φ)

→ Expansion on spherical harmonics series

$$\xi_r = \sum_{\ell \geq |m|}^{\infty} \widetilde{\xi}_{r,n,\ell}(r)\, Y_\ell^m(\theta,\varphi)\, e^{i\sigma t}$$

→ Resolution of the 2D eigenvalue problem

Codes: TOP (Reese et al. 2006) and ACOR (Ouazzani et al. 2012b)

- Through Ray dynamics formalism

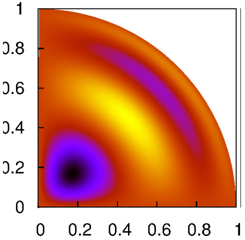
Lignieres & Georgeot 2008, 2009, Pasek et al. 2012, Prat et al. 2016

Acoustic modes in δScuti stars: behaviour with increasing rotation

ex: • Model: 2D polytropic (N=3) • Pulsations: 2D non-perturbative

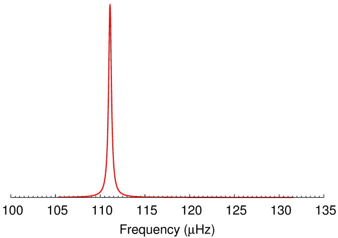
Rotation: $v_{rot} = 0$ km/s

kinetic energy in a meriodional plane



- Spherical symmetry

triplet in the frequency domain



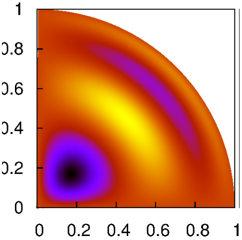
- Degeneracy of the frequencies

Acoustic modes in δScuti stars: behaviour with increasing rotation

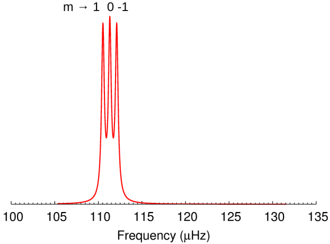
ex: • Model: 2D polytropic (N=3) • Pulsations: 2D non-perturbative

Rotation: $v_{rot} = 15$ km/s

kinetic energy in a meriodional plane



triplet in the frequency domain



• Spherical symmetry

• Lift of degeneracy

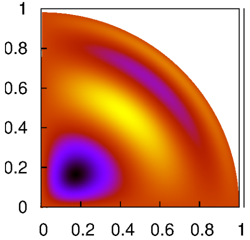
1storder perturbative method $\sigma_{n,\ell,m} = \sigma_{n,\ell}^{\Omega=0} + m \frac{1}{\mathrm{I}_{n,\ell}} \int_0^R K_{n,\ell}(r)\Omega(r)\rho_0 r^2 dr$

Acoustic modes in δScuti stars: behaviour with increasing rotation

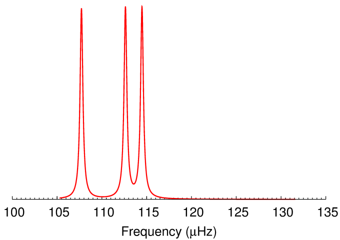
ex: • Model: 2D polytropic (N=3) • Pulsations: 2D non-perturbative

Rotation: $v_{rot} = 52$ km/s

kinetic energy in a meriodional plane



triplet in the frequency domain



- Slight distortion

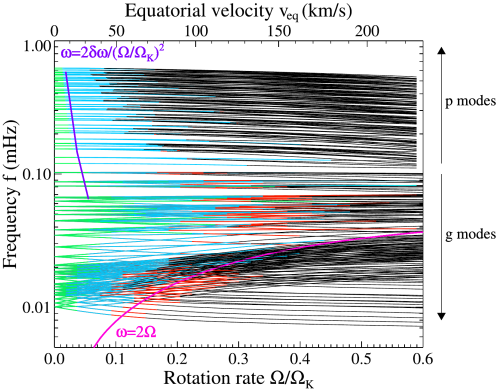
$$p_0 = p_{00} + \epsilon^2 p_2 P_2(\cos\theta)$$

- Assymmetry of the triplets

$$\sigma_{n,\ell,m} = \sigma_{n,\ell}^{\Omega=0} + m\sigma_{n,\ell,1} + \Omega^2 \left(D_{1,n,\ell} + m^2 D_{2,n,\ell}\right)$$

2nd order perturbative methods (Gough&Thompson 1990, Dziembowski&Goode 1992, Suarez&Goupil 2008)

To what extent are the perturbative methods valid ?



Rotation as a small perturbation:

$$\omega_{n,\ell,m} = \omega_{n,\ell}^{\Omega=0} + m\Omega C_{n,\ell,1} + \Omega^2 C_{n,\ell,m,2} + \Omega^3 C_{n,\ell,m,3} \dots$$

Validity domain ? Ballot et al. 2010

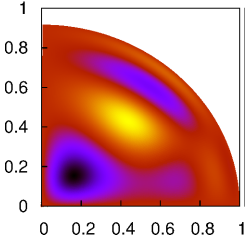
Comparison between complete and 1^{st}, 2^{nd}, and 3^{rd} perturbative methods.

⇒ Asteroseismology based on non-perturbative modelling of pulsations

Acoustic modes in δScuti stars: behaviour with increasing rotation

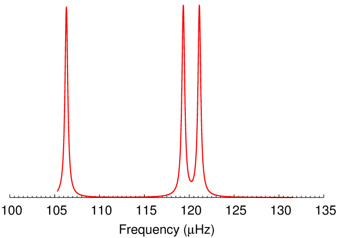
Rotation: $v_{rot} = 142$ km/s

kinetic energy in a meriodional plane



- Mixed symmetry $\ell = 1/\ell = 3$

triplet in the frequency domain

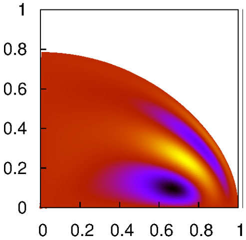


- Assymmetry of the triplets

Acoustic modes in δScuti stars: behaviour with increasing rotation

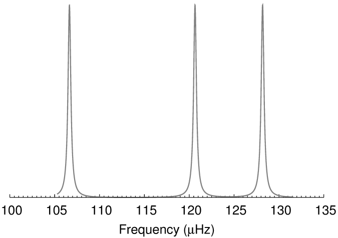
Rotation: $\mathrm{v_{rot}} = 233$ km/s

kinetic energy in a meriodional plane



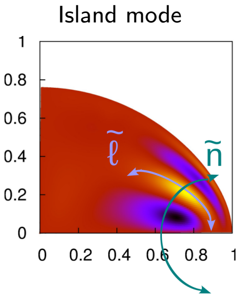
- New symmetry ?

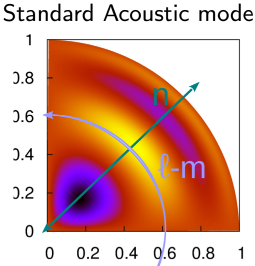
triplet in the frequency domain



- No triplets

Acoustic modes in δScuti stars: new geometry and frequency spectrum





→ Reese et al. 2008

$\tilde{n} = 2n + \eta, \qquad \tilde{\ell} = \dfrac{\ell - |m| - \eta}{2}, \qquad$ with $\eta = (\ell + m) \bmod 2, \qquad$ and m unchanged

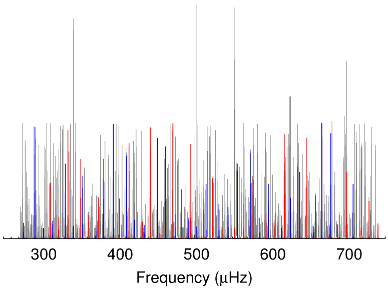
Asymptotic spectra: $\quad \sigma_{\tilde{n},\tilde{\ell},m} = \Delta_{\tilde{n}}\tilde{n} + \Delta_{\tilde{\ell}}\tilde{\ell} + \Delta_m |m| + \alpha$

→ Lignières & Georgeot 2008

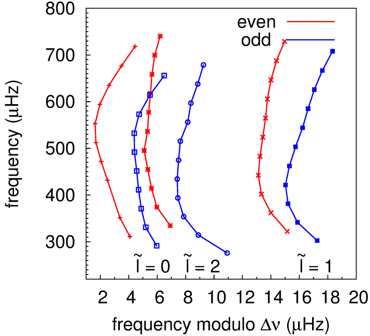
Ray dynamics: $\quad \Delta_{\tilde{n}} = \pi \Big/ \left(\int_a^b \dfrac{ds}{c_s} \right), \quad$ with s being the location on the ray path.

Acoustic modes in δScuti stars: a forest of modes!

Exploration of the seismic spectrum:
2 $M_\odot$, 2.4 $R_\odot$ fully distorted model
evolved until $X_c = 0.35$
Rotation velocity: $\Omega = 80\%\Omega_k$
Roxburgh 2006



$\eta = 0 \rightarrow$ even, $\eta = 1 \rightarrow$ odd



$$\Delta_{\tilde{n}} \quad \text{being compatible with} \quad \pi / \left(\int_a^b \frac{ds}{c_s} \right)$$

Ouazzani et al. 2015 confirms ray dynamics prediction Lignieres & Georgeot 2008

Acoustic modes in δ Scuti stars: Observations

- Can island modes regularity be detected in Observations ?

- Fourier transform of the frequencies with the highest amplitudes
in CoRoT targets Garcia-Hernandez et al. 2009, 2013b,
with *Kepler* Garcia-Hernandez et al. 2013a.

- Automatic search for constant spacing within a tolerance interval
in CoRoT EXO fields Paparo et al. 2016 a,b.→ see Poster #7

- Filtered Autocorrelation adapted from the red giants Mosser&Appourchaux 2009
in 1900 δ Scuti stars of CoRoT EXO fields (Michel et al. in prep).

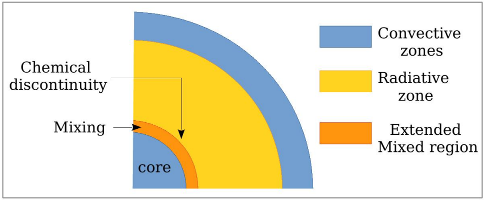
- Is the spacing related to a structural quantity ?

- Independent determination of mass and radius from binarity + Roche Model
or interferrometry Garcia-Hernandez et al. 2015
⇒ mean density for 7 δScuti stars with seismology (MOST, CoRoT or *Kepler*)

→ see also Juan Carlos Suarez's talk

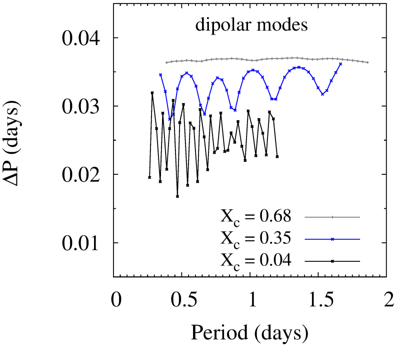
Seismology of γ Doradus stars based on gravito-inertial modes

gravity modes → equally spaced in period ⇒ **Period spacing ΔP**



★ Properties of convective cores

- Mean value and shape ⇒ evolution



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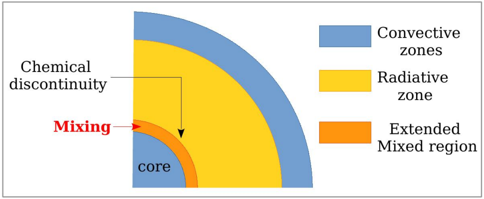

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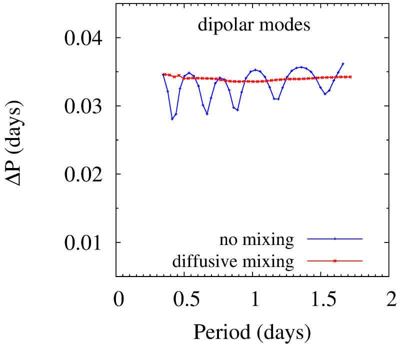
- Mean value and shape ⇒ evolution
- Shape ⇒ mixing above the core



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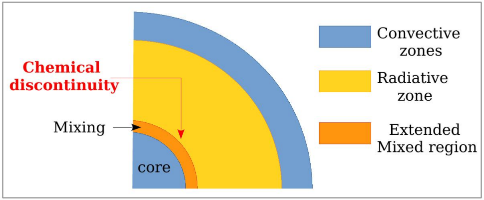
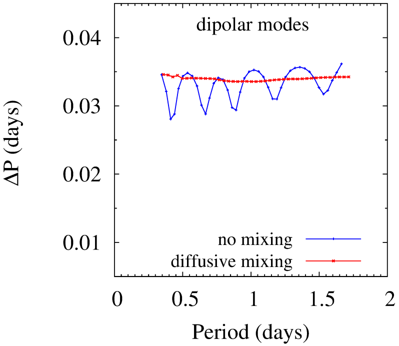


⋆ Properties of convective cores

- Mean value and shape ⇒ evolution
- Shape ⇒ mixing above the core
- Pattern ⇒ location of the discontinuity



Th: Miglio et al 2008
Obs: Saio et al. 2015, Murphy et al. 2016

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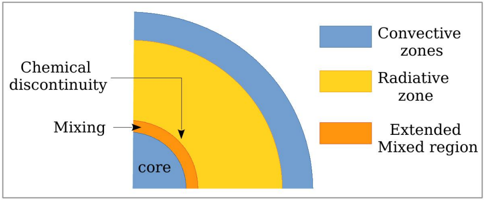
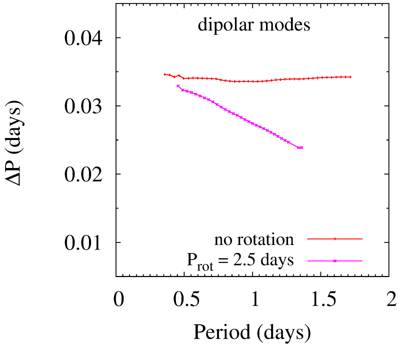


⋆ Effect of internal rotation ?

- Global linear trend on the ΔP ⇒ slope

- Little effect on the excitation range ~ 25-40 radial orders maximum



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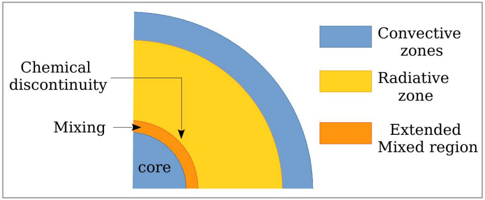

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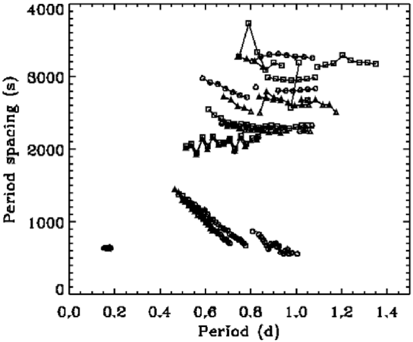

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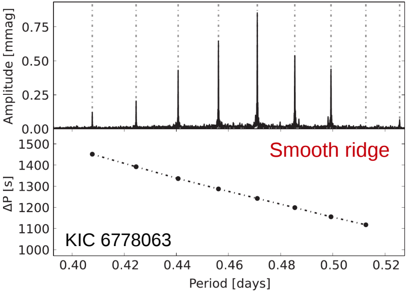
Asteroseismology of fast rotating γ Dors with *Kepler*

Kepler: the game changer! Van Reeth et al. 2015, 2016

Detection of dozens of period spacing ridges in γDoradus stars.



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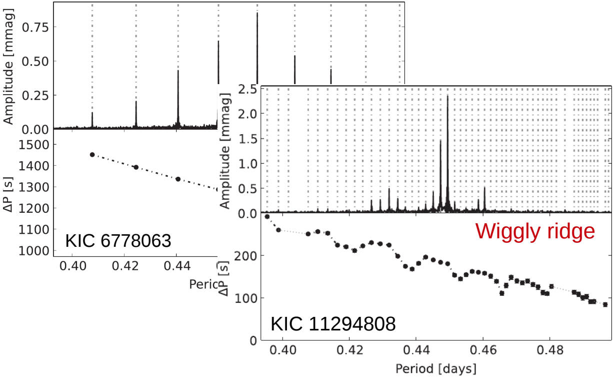

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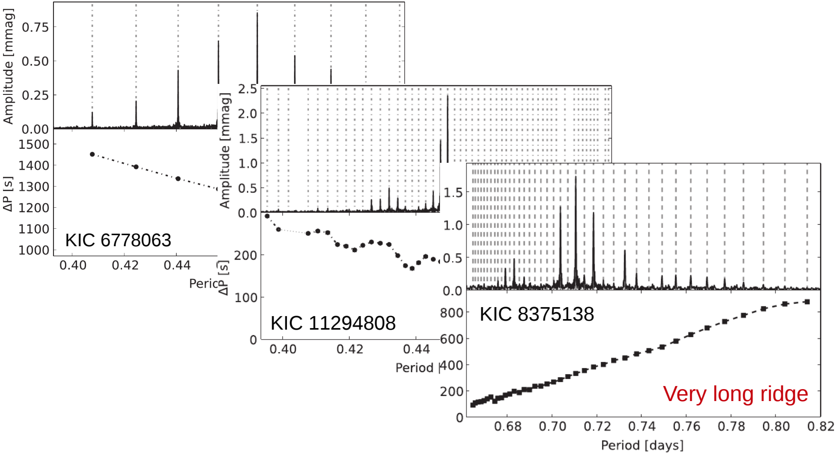

Modelling γ Dor gravito-inertial modes

→ Coriolis force dominates, centrifugal distortion negligible (Ballot et al. 2010)

→ 1D structure + Non-perturbative modelling of pulsations.

- ⋆ **Traditional approximation of Rotation (TAR)** Lee & Saio 1987a

Hypotheses:

- Solid body rotation, • Spherical symmetry, • Cowling approximation,
- Neglects horizontal component of the angular velocity vector

Pulsations Equations separable:
in terms of a radial part + the Hough functions $\ell(\ell+1) \rightarrow \lambda$

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⋆ **Asymptotic relation including the TAR** Townsend 2003b

Hypotheses: • All of the above • $n >> \ell$ → JWKB analysis

$$P_{co}(n) = \frac{2\pi^2(n+\frac{1}{2})}{\sqrt{\lambda_{\ell,m,s(n)}}\int_{r_0}^{r_1}\frac{N}{r}dr}$$

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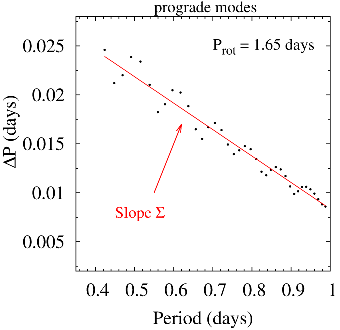
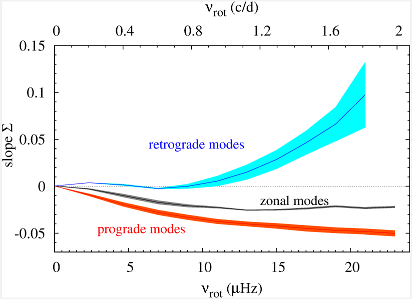
★ **Non-perturbative computations** Lignieres et al. 2006, Reese et al. 2006, Ballot et al. 2010, Ouazzani et al. 2012b, 2015

A New asteroseismic diagnostic for rotation in γ Dors

Theoretical exploration of period spacing behaviour up to fast rotation
Based on Non-perturbative modelling of pulsations Ouazzani et al. sub.



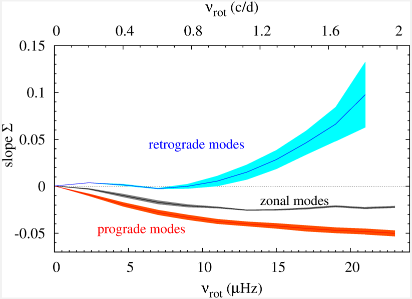
Univoque relation between Σ,
slope of the ΔP ridge and rotation
for all the MS γ Doradus stars.



A New asteroseismic diagnostic for rotation in γ Dors

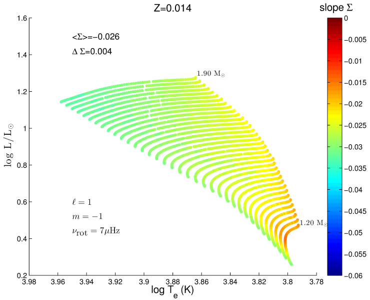
Theoretical exploration of period spacing behaviour up to fast rotation
Based on Non-perturbative modelling of pulsations   Ouazzani et al. sub.

Univoque relation between Σ, slope of the ΔP ridge and rotation **for all the MS γ Doradus stars**.



Σ does not depend on:

- Centrifugal distorsion
- Metallicity
- Type of mixing above the CC



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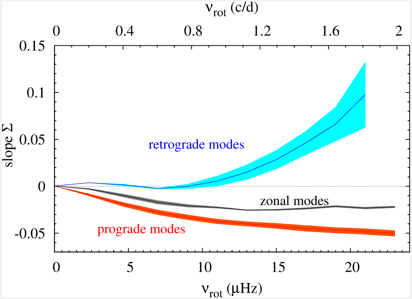
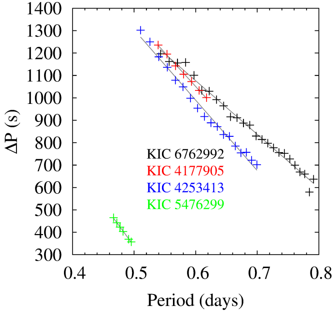
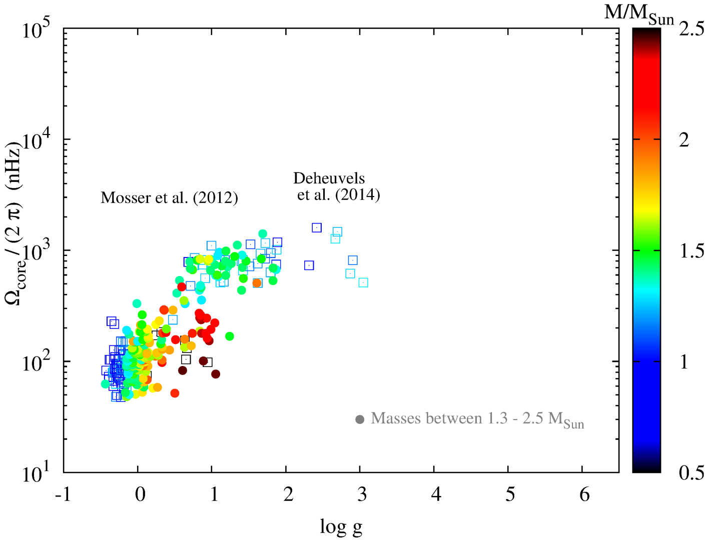


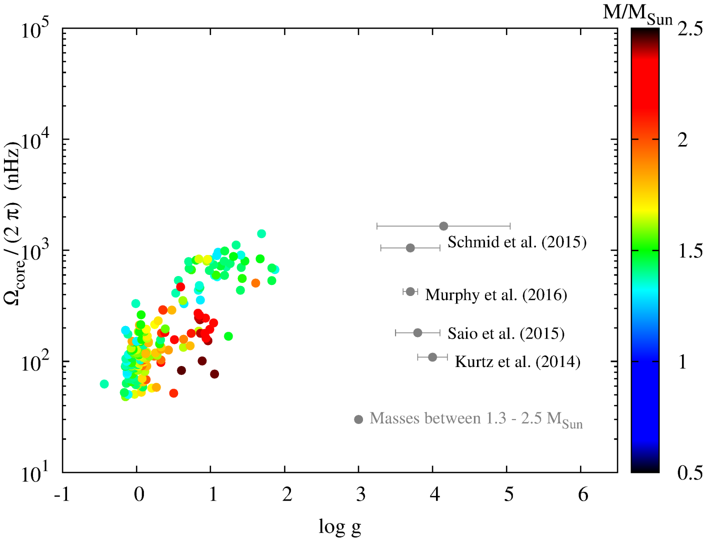


$\Rightarrow$ **Internal rotation rates for 4 Kepler γ Dors**

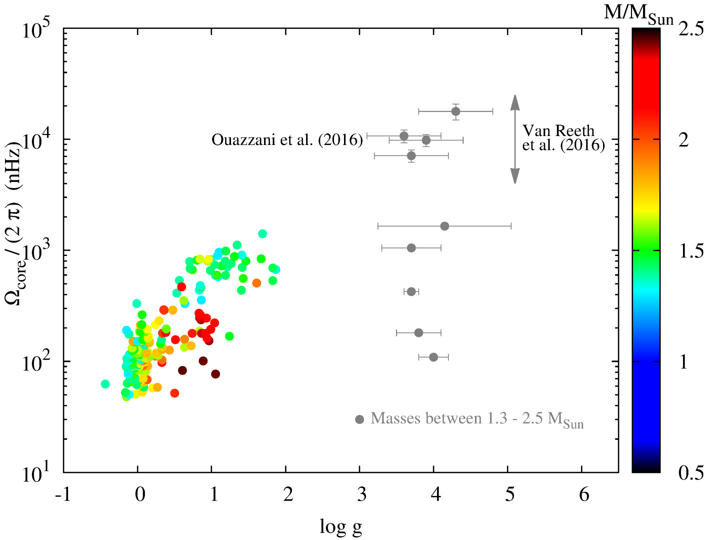
Rotation of γ Dors in the stellar evolution context



Rotation of γ Dors in the stellar evolution context



Rotation of γ Dors in the stellar evolution context



Asteroseismology of rapidly rotating A-stars: We're getting there !

- Perturbative methods not valid, non-perturbative methods still not optimized for seismic modelling...
- This will be possible thanks to seismic diagnostics based on non-perturbative modelling.

δ Scuti stars

- The problem of mode ID remains for the more moderate pulsators,
- For very fast rotating stars, new organization of the frequency spectra. $\Rightarrow$ Asteroseismology based on Island modes regularities.

γ Doradus stars

- We now can use a new seismic diagnostic to infer internal rotation

$\rightarrow$ Diagnostic on the transport of angular momentum from MS to RGB