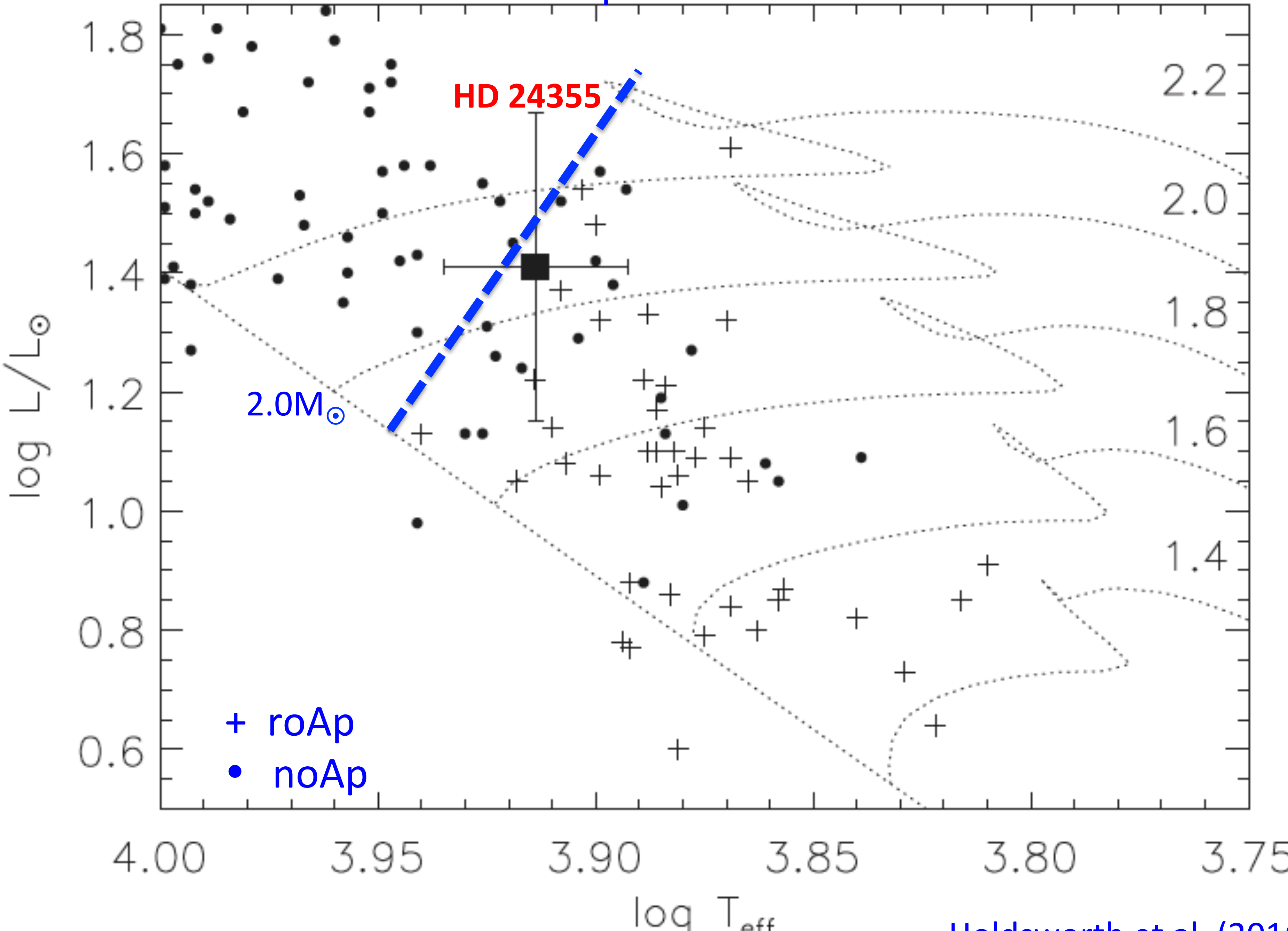
# Models for the amplitude-phase modulations of the peculiar roAp star HD 24355

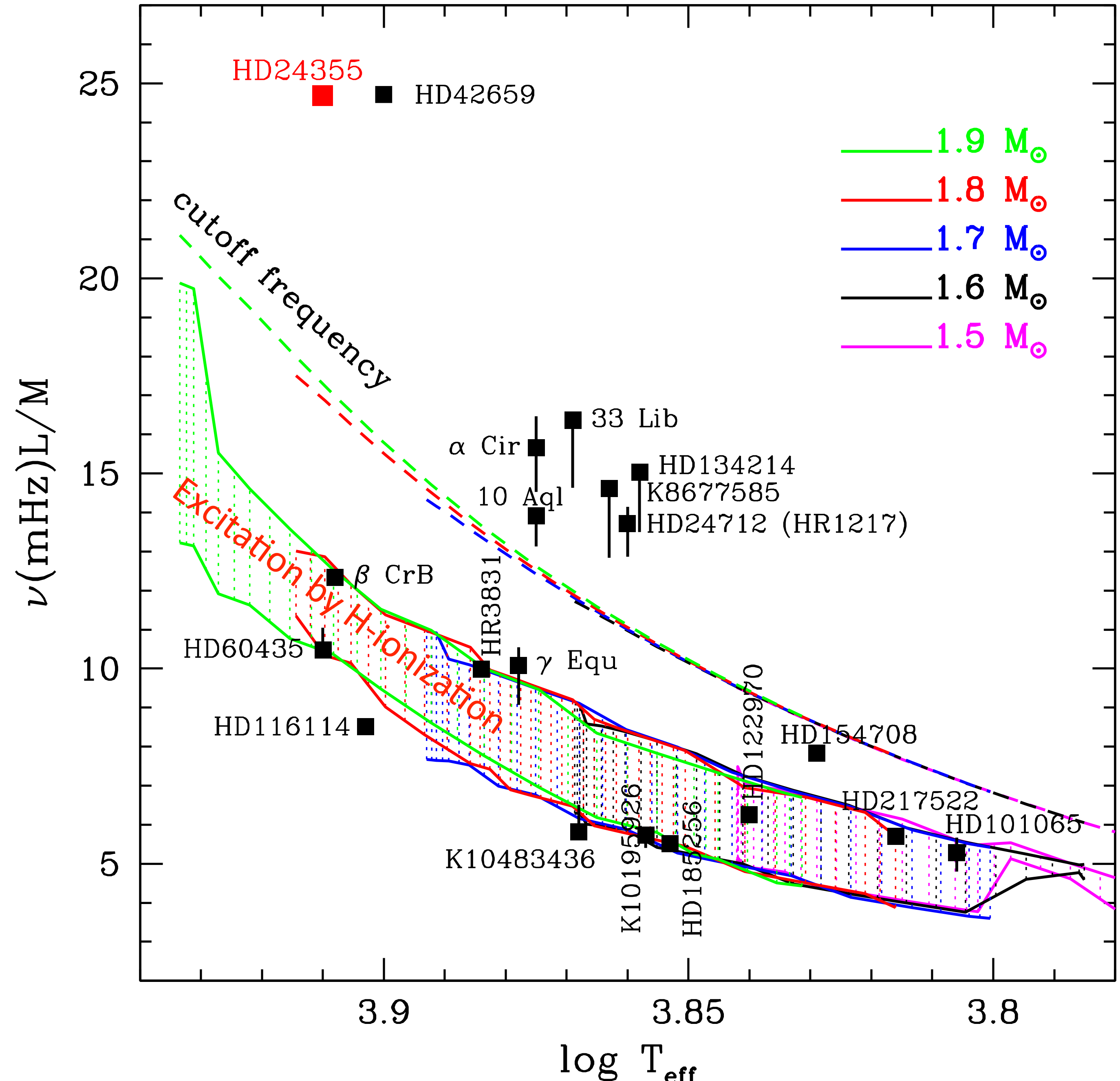
Hideyuki Saio (Tohoku University, Sendai, Japan)

Daniel Holdsworth, Don Kurtz, et al. (2016; MNRAS)
"HD24355 observed by the Kepler K2 mission:
a rapidly oscillating Ap star pulsating
in a distorted quadrupole mode"

# Ap stars



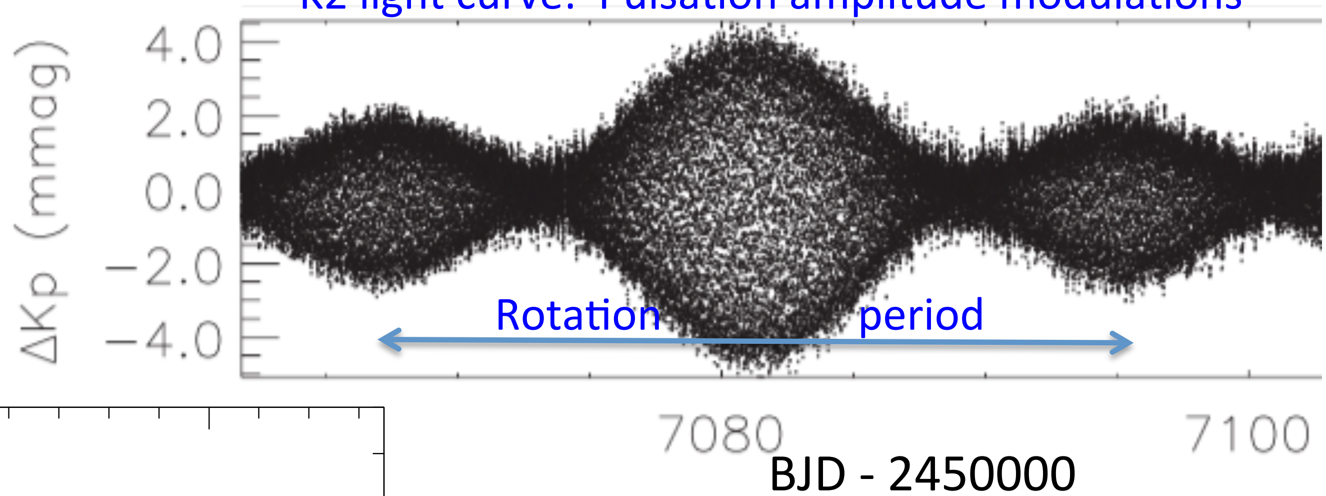

Holdsworth et al. (2016)




Holdsworth et al. (2016)

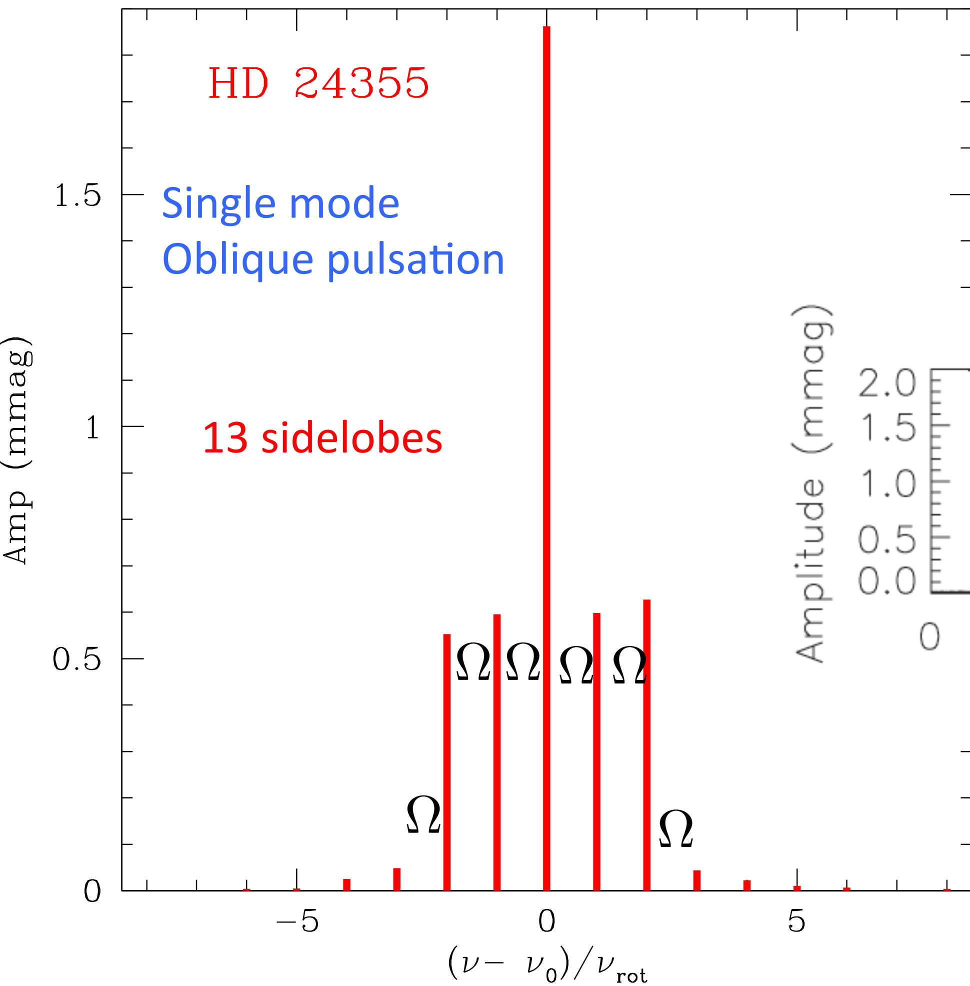
# HD 24355

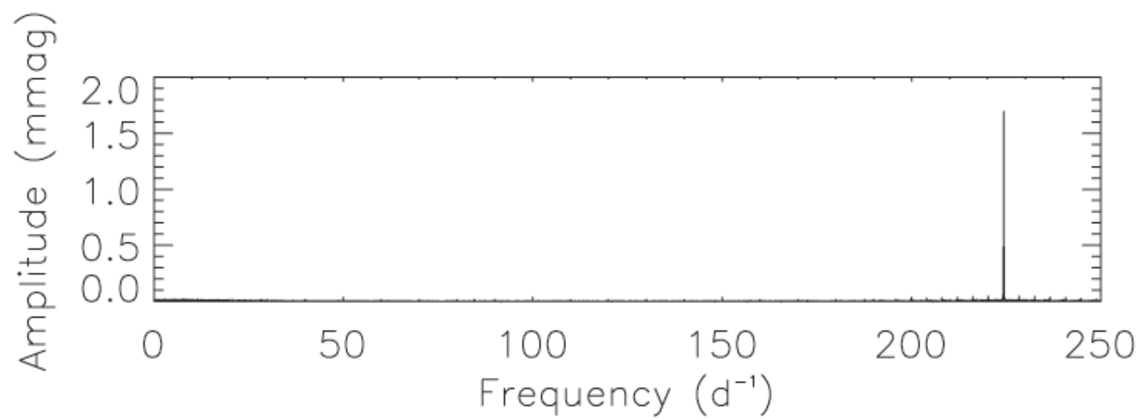
K2 light curve: Pulsation amplitude modulations




Rotation period = 27.9 days

$V \sin i \leq 3.5 km/s \rightarrow i < 50°$




Single mode pulsator

224.31 c/d (2.60 mHz; P = 6.4 min)

Radial order ≈ 47

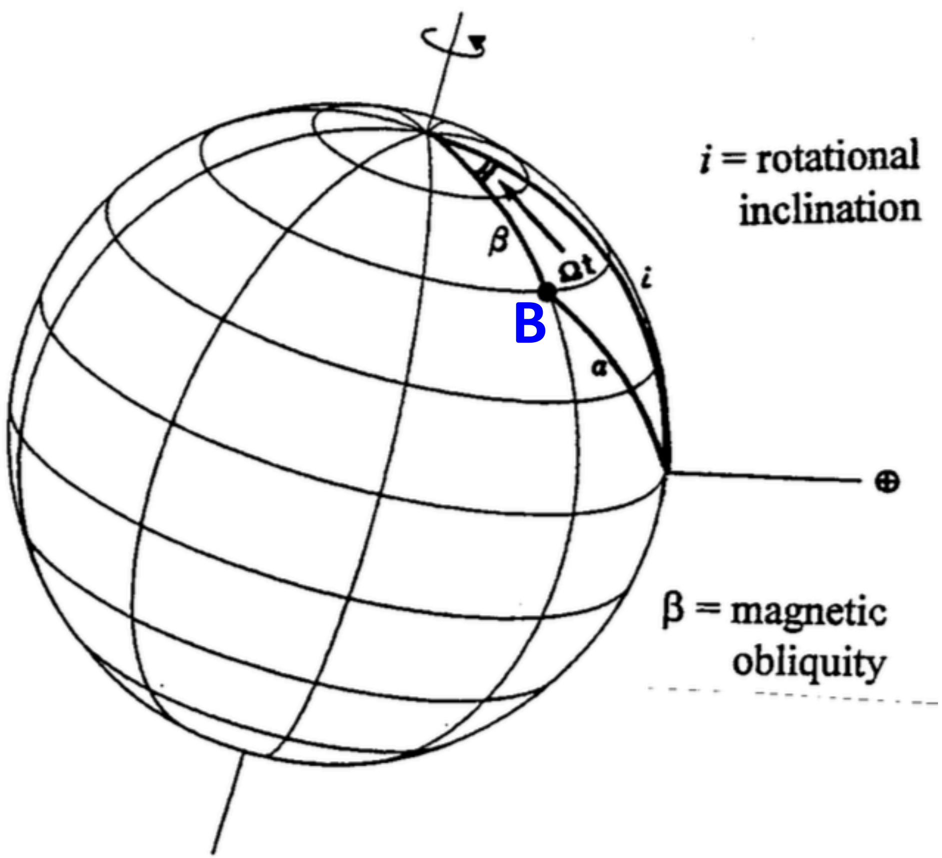
Holdsworth et al. (2016)

# Oblique pulsation (Kurtz 1982)

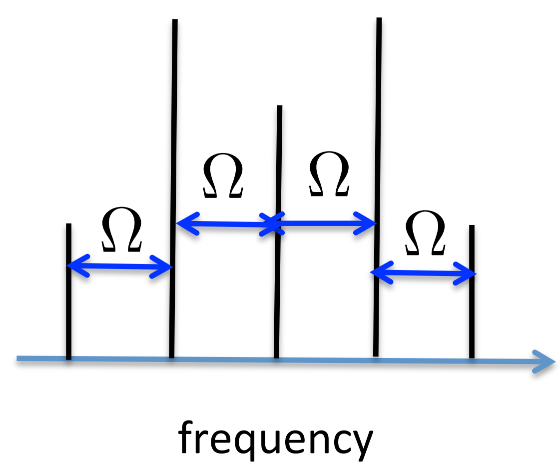
$$\Delta L \propto e^{i\sigma t} Y_\ell^0(\theta_B, \phi_B)$$

$$\propto e^{i\sigma t} \sum_{m=-\ell}^{\ell} d^\ell(i) d^\ell(\beta) Y_\ell^m(\theta_L, \phi_L) e^{im\Omega t}$$

L = Line of sight

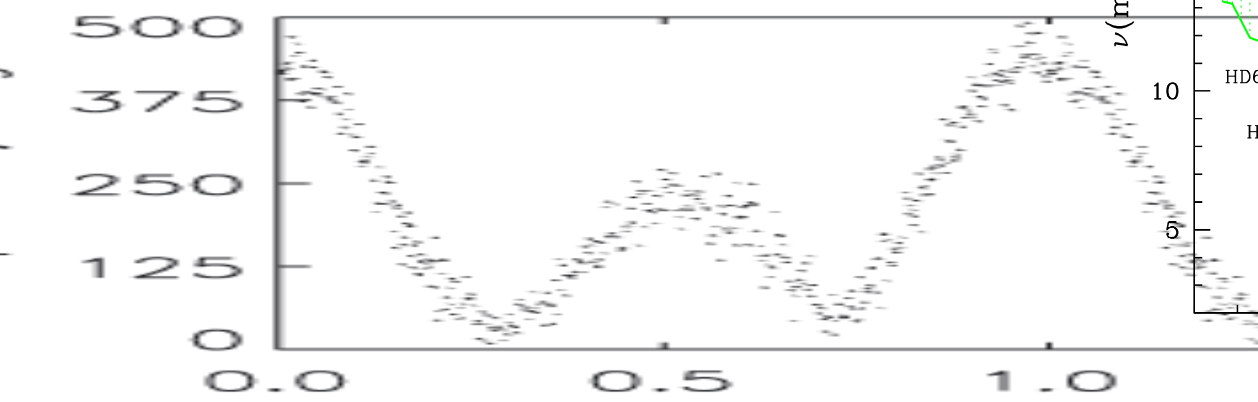


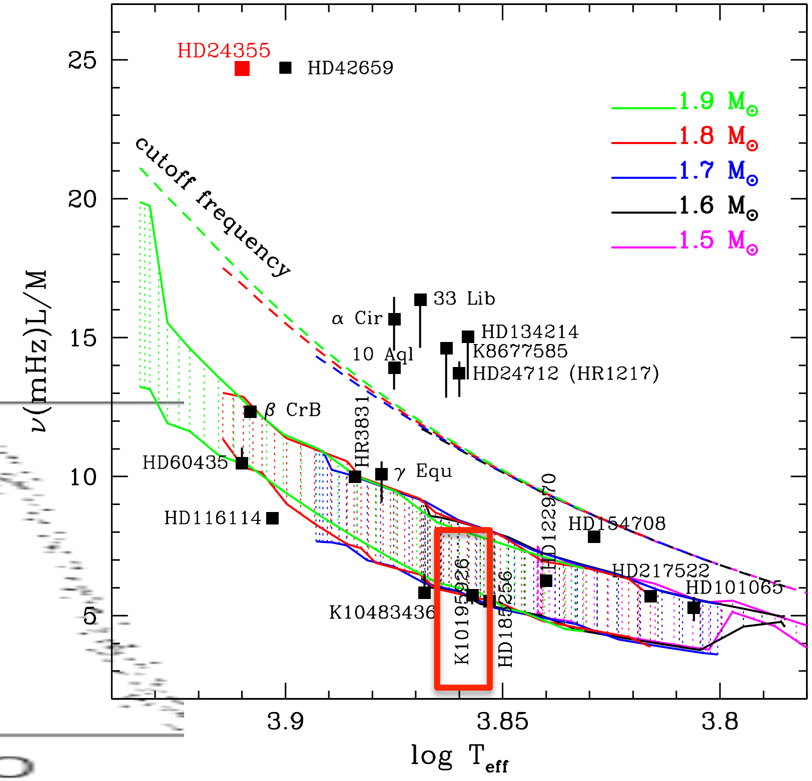

$(2\ell + 1)$ frequencies separated by $\Omega$

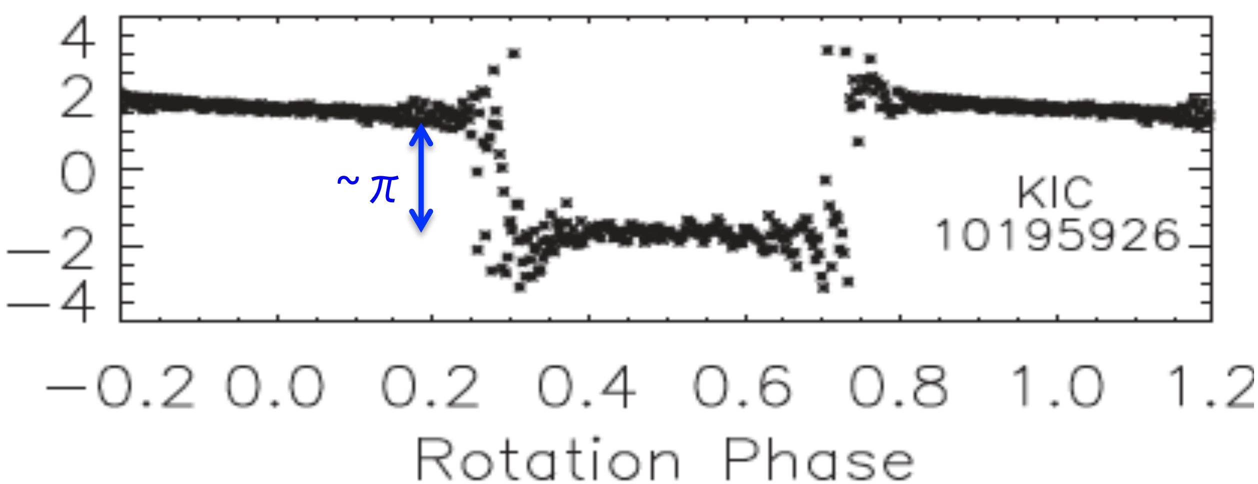


A typical oblique pulsation

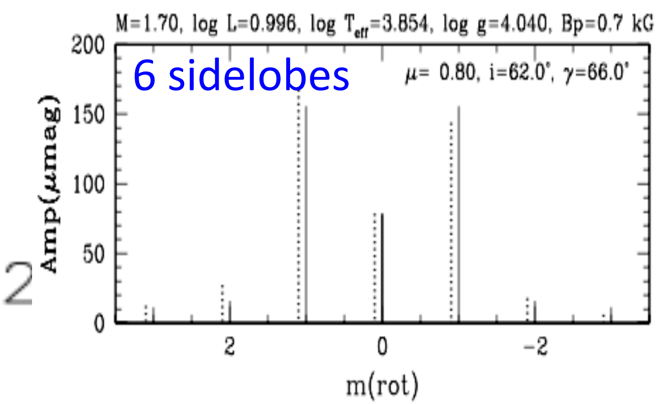
KIC 10195926



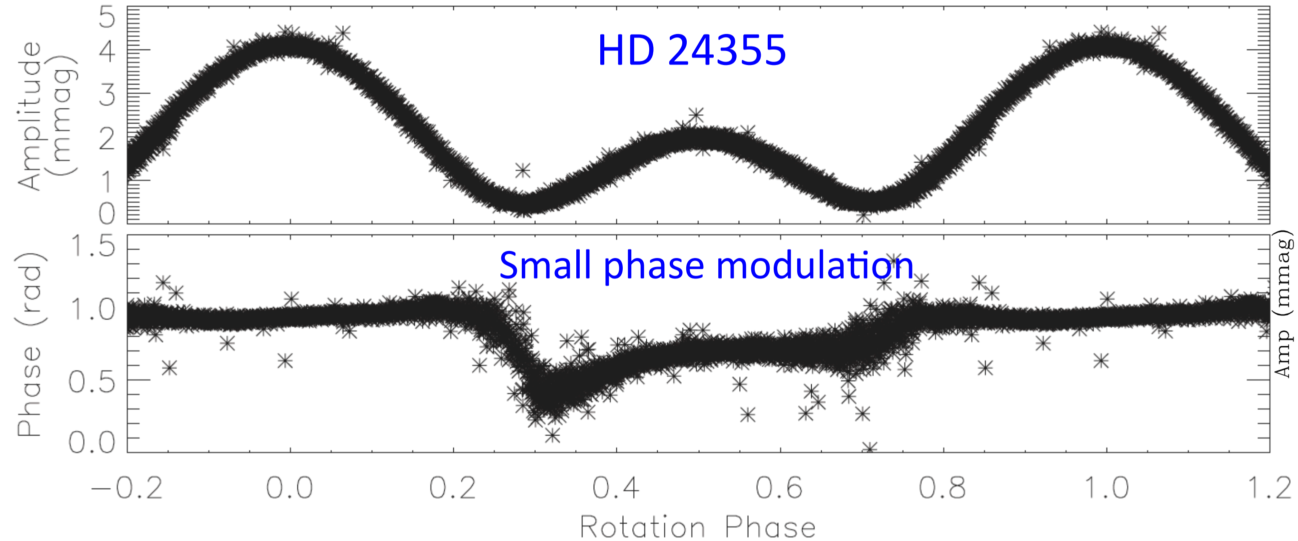



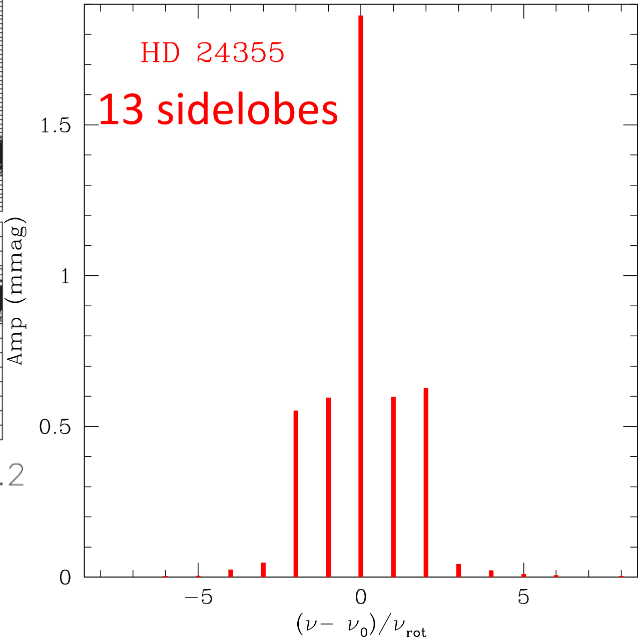



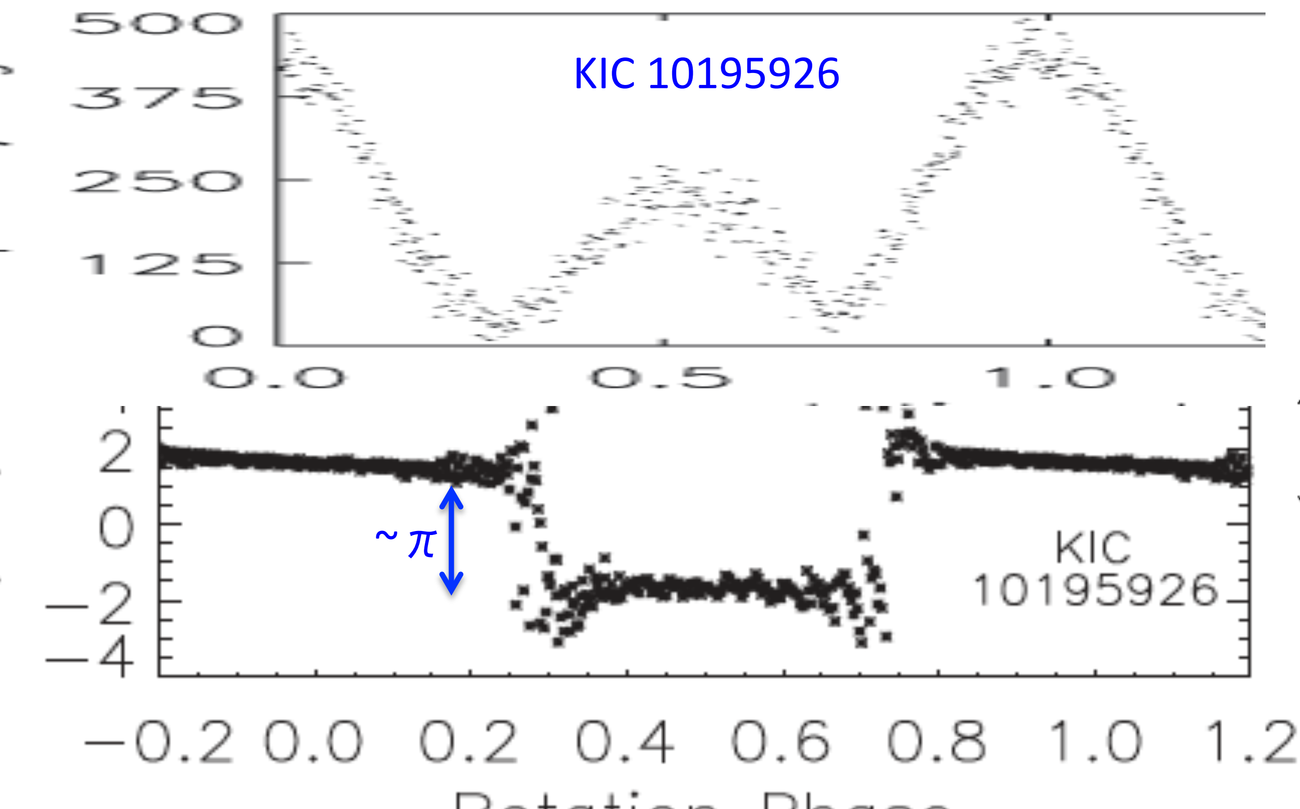

Nearly pure dipole mode;
0.7 kG



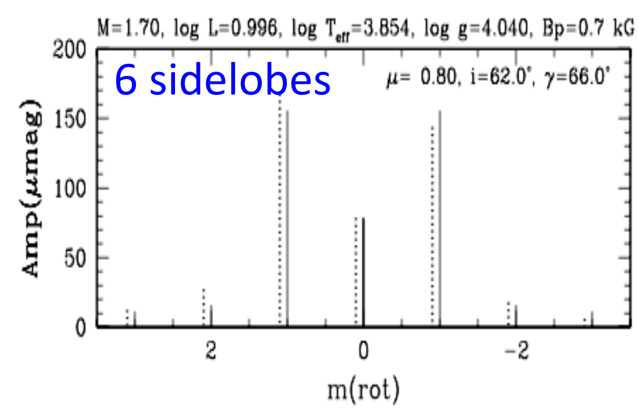

Kurtz et al.(2011)




KIC 10195926
Nearly pure dipole mode;
0.7 kG



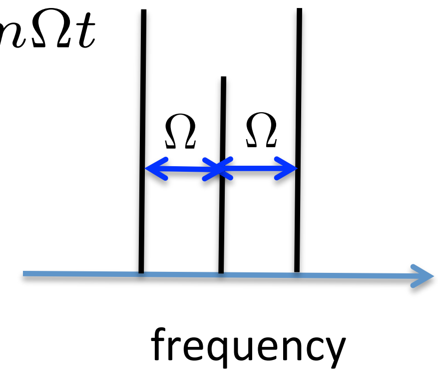

Kurtz et al.(2011)

# Oblique pulsation (Kurtz 1982)

$$\Delta L \propto e^{i\sigma t} Y_\ell^0(\theta_B, \phi_B)$$

$$\propto e^{i\sigma t} \sum_{m=-\ell}^{\ell} d^\ell(i) d^\ell(\beta) Y_\ell^m(\theta_L, \phi_L) e^{im\Omega t}$$

L = Line of sight



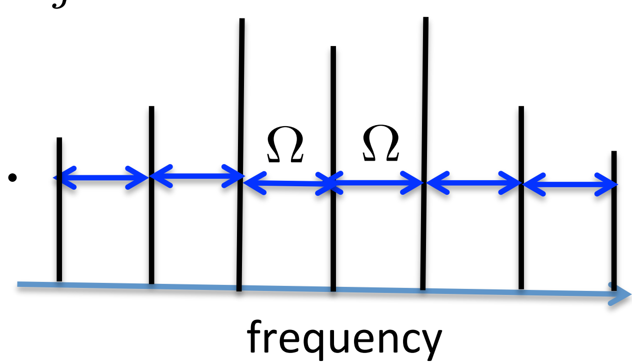

Magnetic couplings

$$\Delta L \propto e^{i\sigma t} \sum_j A_j Y_{\ell_j}^0(\theta_B, \phi_B)$$

Additional components

$$\propto e^{i\sigma t} \sum_j A_j \sum_{m=-\ell_j}^{\ell_j} d^{\ell_j}(i) d^{\ell_j}(\beta) Y_{\ell_j}^m(\theta_L, \phi_L) e^{im\Omega t}$$

$\ell_j = 1,\ 3,\ 5,\ \ldots$ or $\ell_j = 0,\ 2,\ 4\ \ldots$




$A_j$ = complex amplitude of j component

# Pulsation equations are solved to obtain $A_j$

Ideal MHD approximation: $$\frac{\partial \boldsymbol{B}'}{\partial t} = \nabla\times\left(\frac{d\boldsymbol{\xi}}{dt}\times\boldsymbol{B}_0\right)$$

Linearized equation of motion:

$$\frac{d^2\boldsymbol{\xi}}{d^2 t} = \frac{\rho'}{\rho^2}\frac{dp}{dr}\boldsymbol{e}_r - \frac{1}{\rho}\nabla p' + \boxed{\frac{1}{4\pi\rho}(\nabla\times\boldsymbol{B}')\times\boldsymbol{B}_0}$$

Perturbation of Lorentz force

$$\frac{\left|\frac{1}{4\pi\rho}(\nabla\times B')\times B_0\right|}{\left|\frac{\nabla p'}{\rho}\right|} \sim \left(\frac{V_A}{C_S}\right)^2$$

$$V_A = \frac{B}{\sqrt{4\pi\rho}}$$ Alfven velocity

Magnetic-acoustic coupling occurs where $V_A \sim C_S$

$$f' = \sum_j f'_{\ell_j} Y^0_{\ell_j}(\theta_B, \phi_B)$$

$$B'_{h,\ell_j}, \quad \left(\xi_h - \frac{p'}{\rho\sigma^2}\right)_{\ell_j}, \quad p'_{\ell_j}/\rho, \quad \xi_{r,\ell_j}, \quad \delta L_{rad,\ell_j}, \quad \delta s_{\ell_j}$$

magnetic ($B'_{h,\ell_j}$, $\left(\xi_h - \frac{p'}{\rho\sigma^2}\right)_{\ell_j}$) — mechanical ($p'_{\ell_j}/\rho$, $\xi_{r,\ell_j}$) — thermal ($\delta L_{rad,\ell_j}$, $\delta s_{\ell_j}$)

# How I solved equations

Magnetic variables
R > r > ~0.95R

Mechanical & thermal variables
R > r > 0

Surface $\nabla \times B' = 0$ $\delta p \to 0$

$C_S \ll V_A$

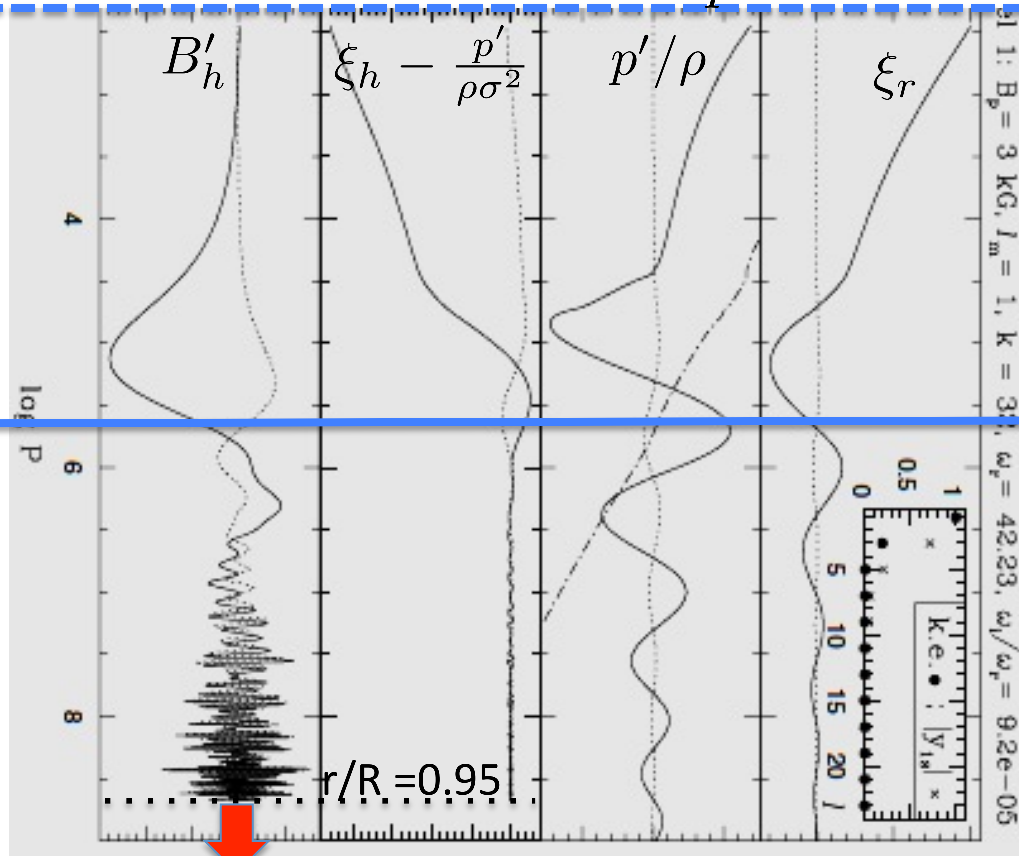
$C_{\rm slow} \sim C_S$

$\cdots$pulsation

magnetic-acoustic wave

$C_S = V_A$

coupling




$C_S \gg V_A$

$C_{\rm slow} \sim V_A \ll C_S$

$C_{\rm fast} \cdots$ pulsation

Center

Slow wave will be dissipated
→ pulsation energy loss
(Roberts & Soward 1983)

$\sigma$ is complex even in adiabatic pulsation

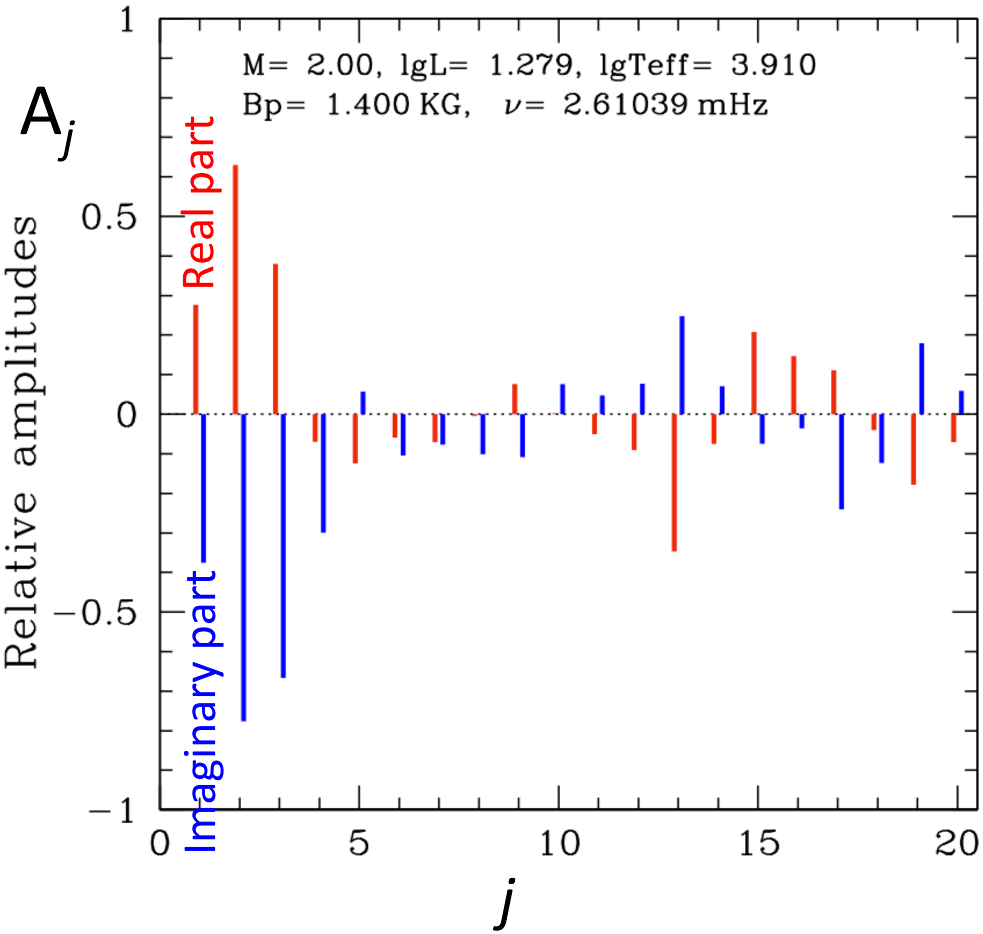
$$V_A = \frac{B}{\sqrt{4\pi\rho}}$$

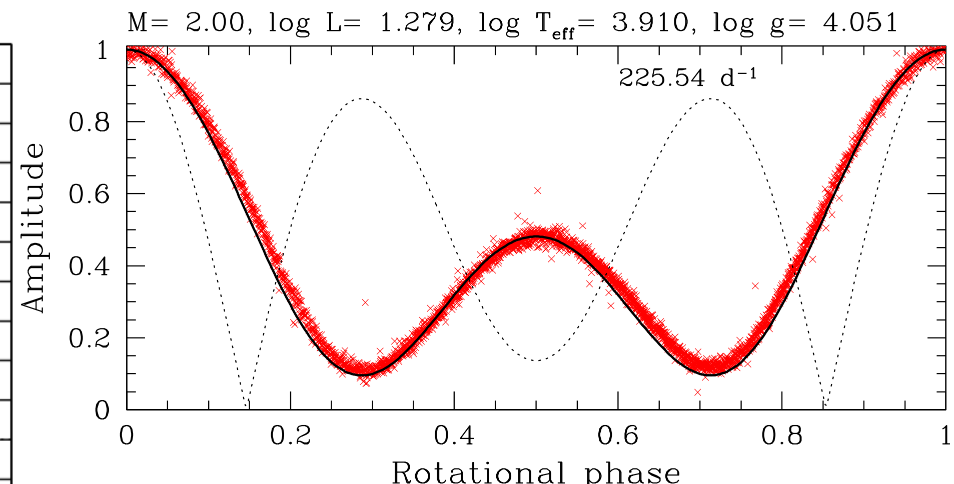
$$C_{\rm fast} = \sqrt{C_S^2 + V_A^2}$$

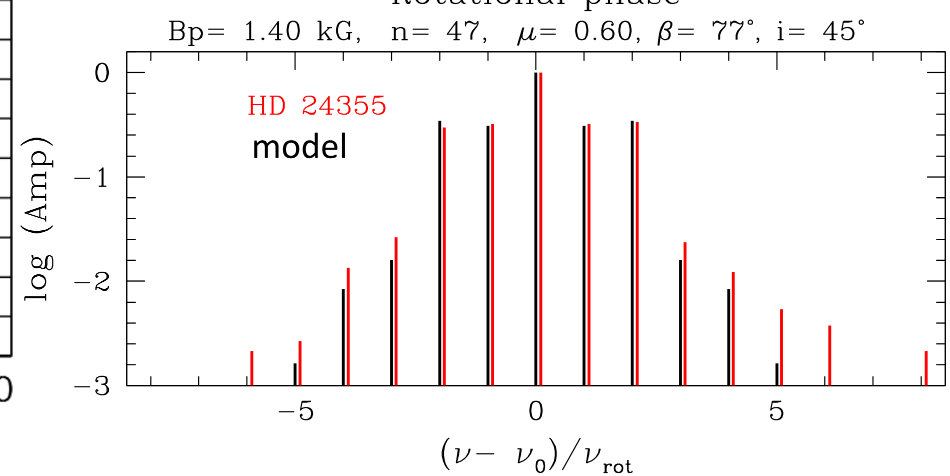
$$C_{\rm slow} = \frac{C_S V_A}{\sqrt{C_S^2 + V_A^2}}$$

$$\Delta L \propto e^{i\sigma t}\sum_{j=1}^{20} A_j Y_{\ell_j}^0(\theta_B,\phi_B) \quad \propto \sum_{j=1}^{20} A_j \sum_{m=-\ell_j}^{\ell_j} d^{\ell_j}(i) d^{\ell_j}(\beta) \times Y_{\ell_j}^{m}(\theta_L,\phi_L) e^{i(\sigma+m\Omega)t}$$

$$\ell_j = 0,\ 2,\ 4,\ 6,\ \ldots, 38$$



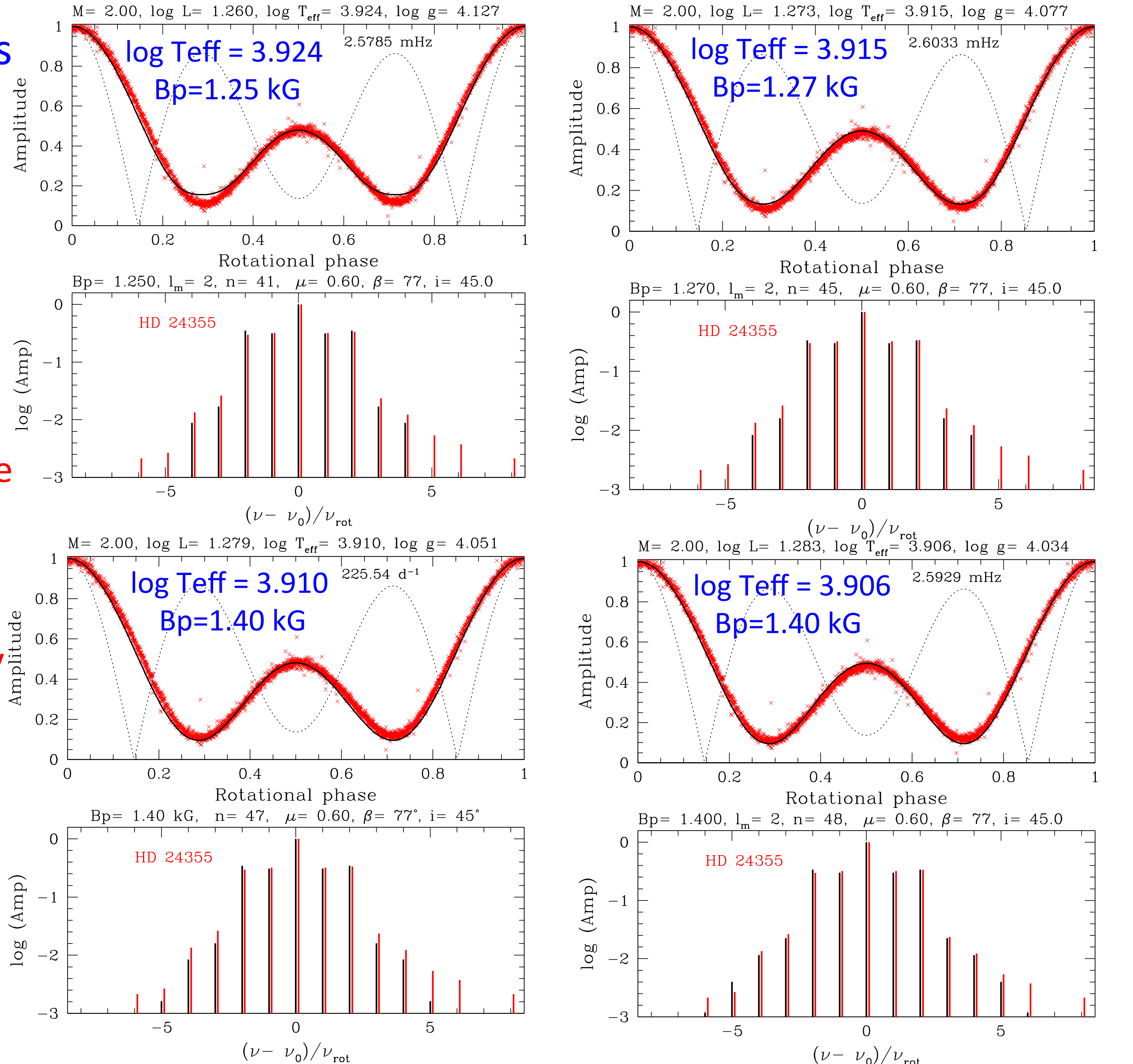



## 2.0$M_\odot$ models

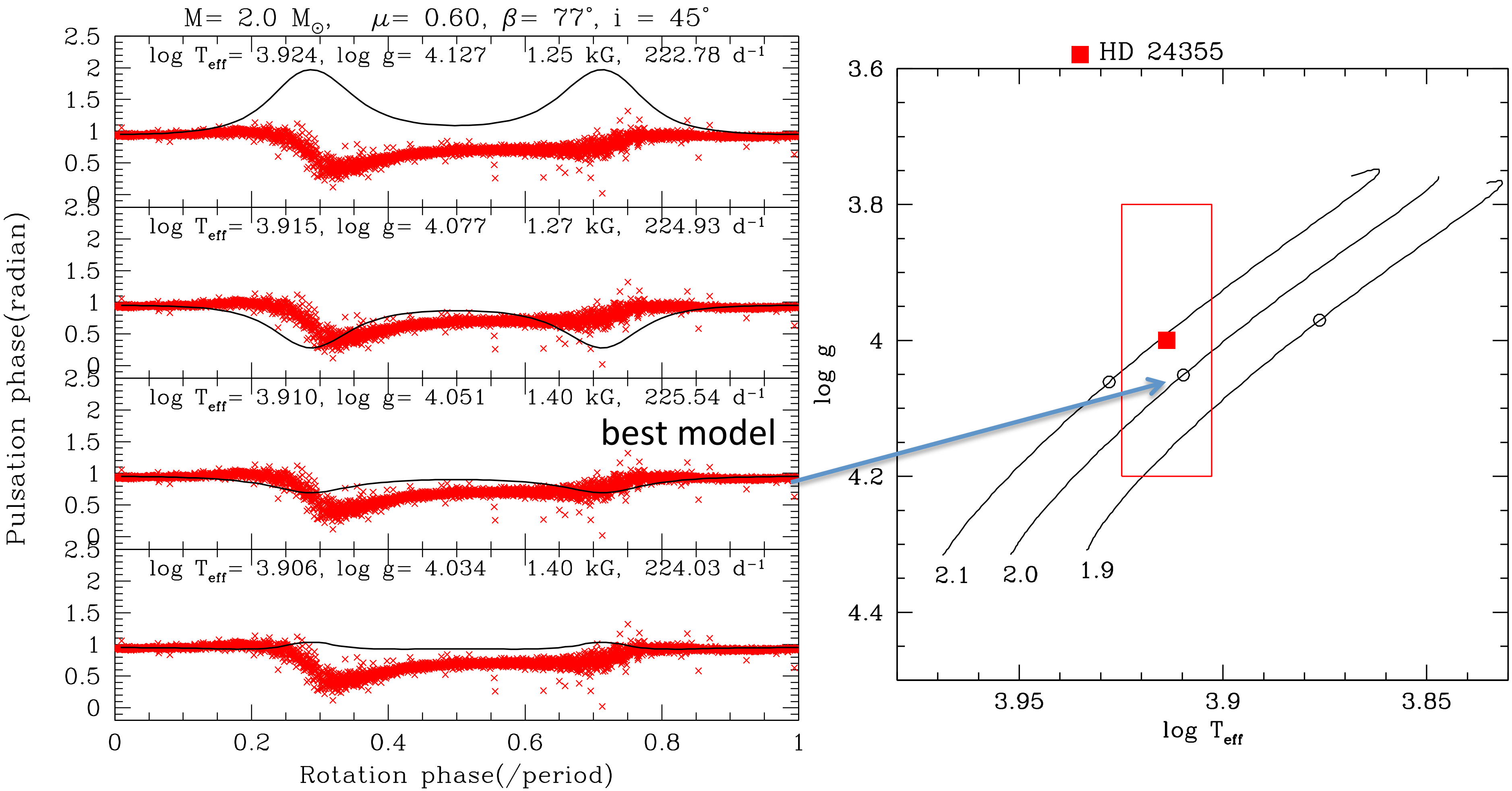
i = 45 deg

beta = 77 deg

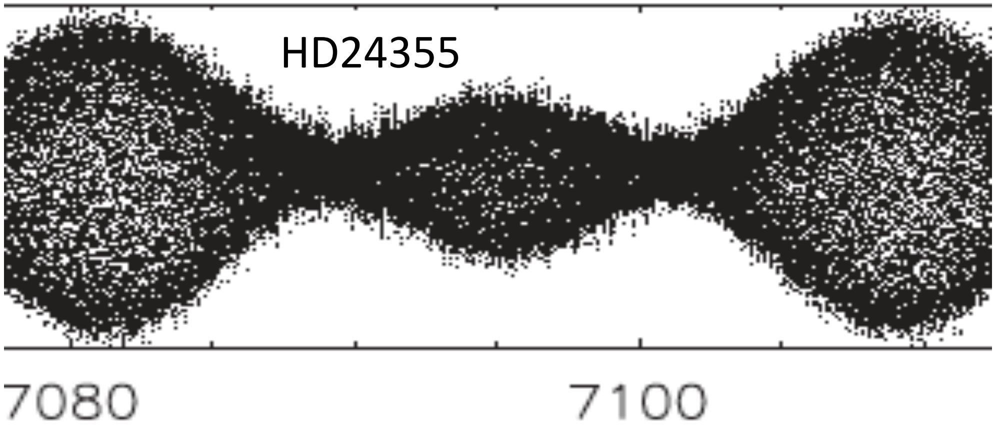
Similar amplitude modulations are obtained at various Teff on the evolutionary track of a given mass



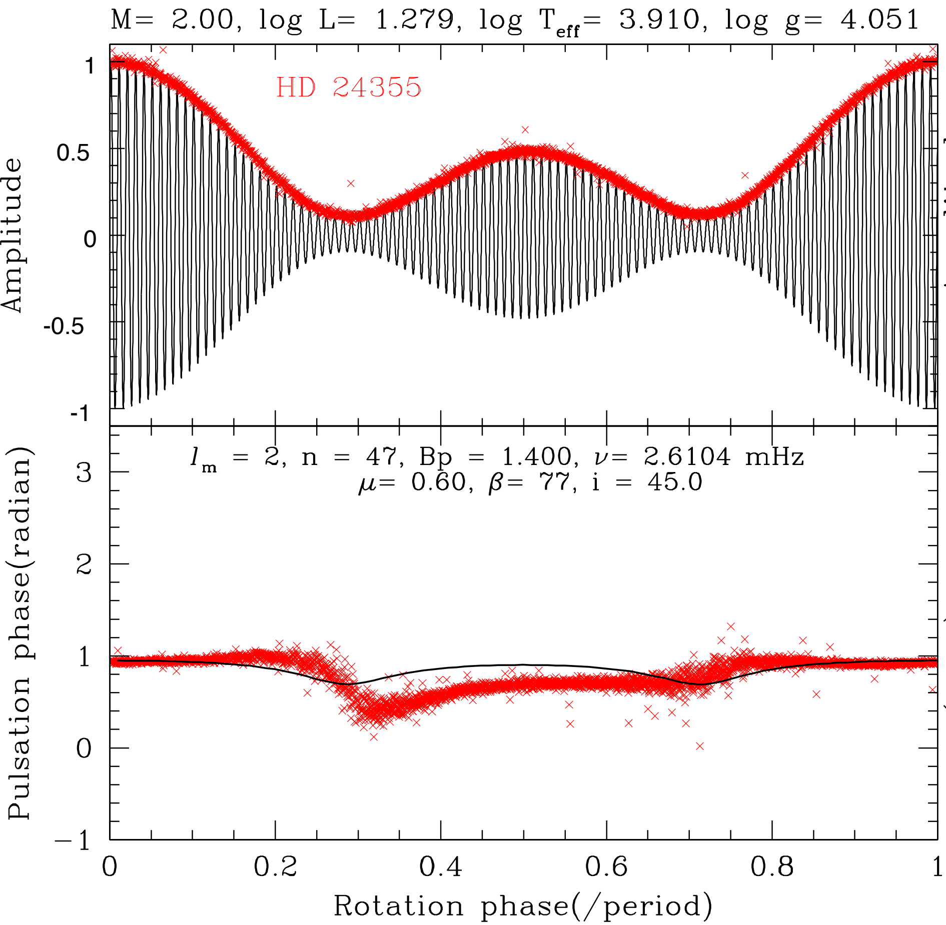
# Phase modulation varies gradually along evolution track

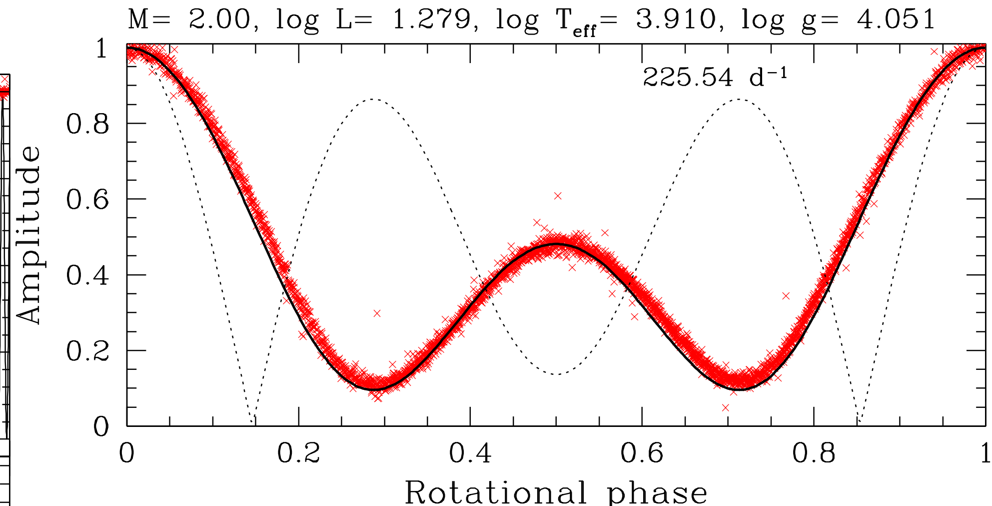


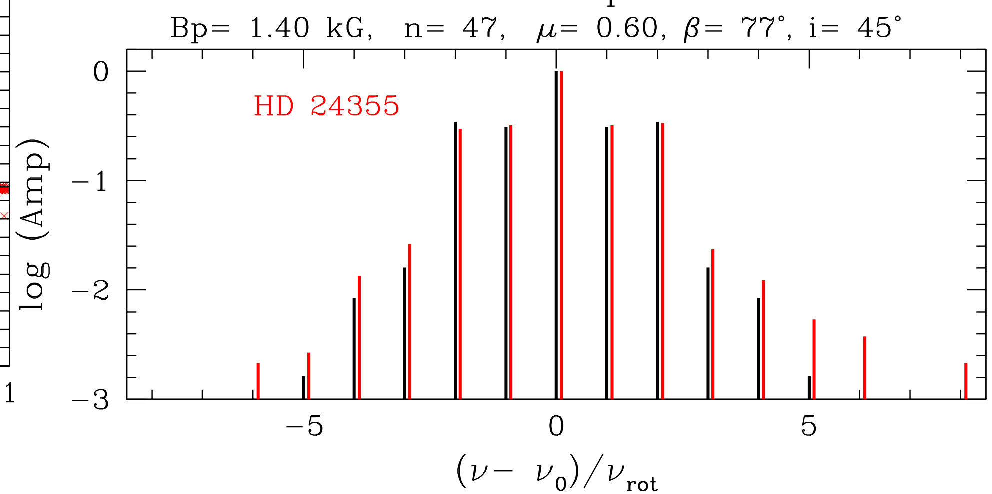
# Best model of HD 24355

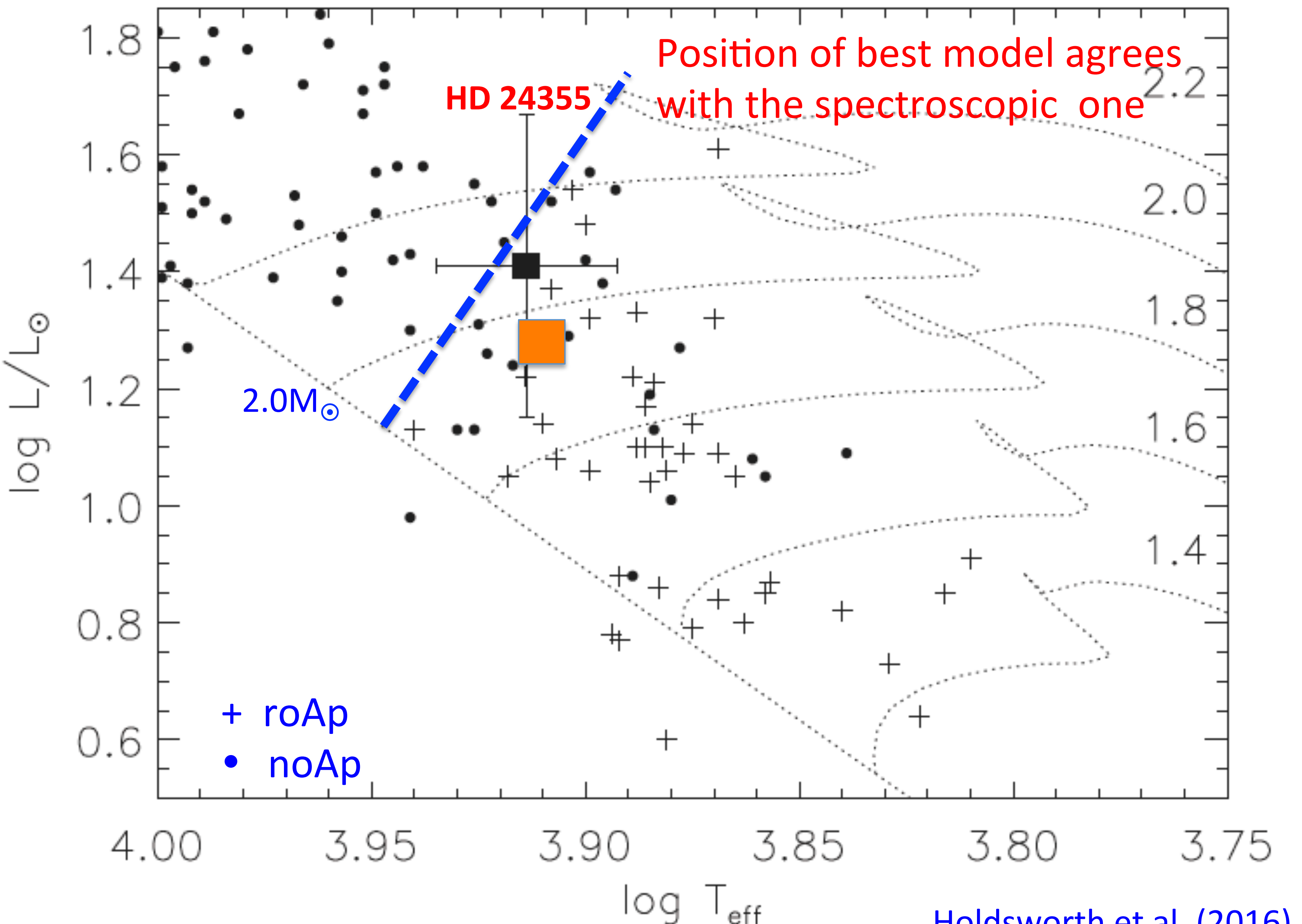



## M=2.0 Bp= 1.40 kG



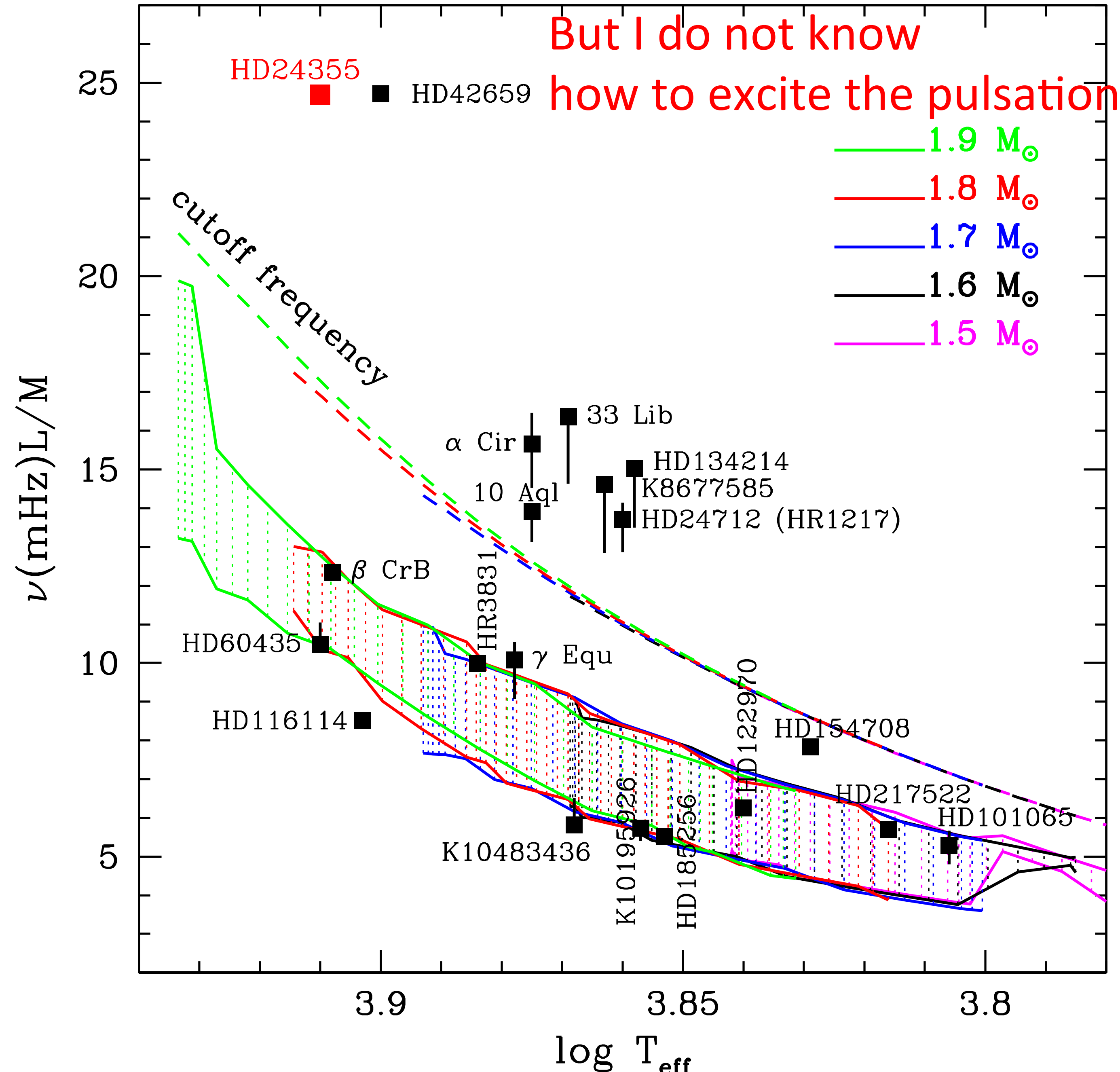





Holdsworth et al. (2016)

But I do not know how to excite the pulsation




Holdsworth et al. (2016)

Special thanks to Don Kurtz for introducing me to many interesting pulsating stars